Bose-Einstein Condensation
II. Some exotic states

Sadhan K. Adhikari

IFT - Instituto de Física Teórica

UNESP - Universidade Estadual Paulista

São Paulo, Brazil

Topics

- I. Binding superfluid bosons and boson-fermion mixture without a trap in 3D: Quantum ball (QB)
- Attractive 2-body and repulsive 3-body interactions, Lee-Huang-Yang correction
- Non-dipolar and dipolar atoms
- Numerical and variational solution of mean-field model
- Quasi-elastic collision of QBs
- II. Stable dark soliton in dipolar BEC in 1D
- III. Unitarity and beyond-mean-field model: Vortex lattice
- Future perspectives

Why study trapless 3D BEC quantum ball?

- The study is expected to be more universal being solely controlled by the atomic interactions, no effect of binding trap.
- New studies: collision, multipole oscillation, Josephson tunneling, interference, vortex formation, spontaneous symmetry breaking, self trapping etc.

Three-body interaction K_3

- At small densities the effect of K_3 is small and is usually neglected
- The quantum two-body interaction is related to the two-body *t* matrix/scattering length *a*
- The three-body term K_3 is related to the three-body *t* matrix and is in general complex, the imaginary part corresponding to a loss of atoms due to molecule formation.
- A very small repulsive (with real part positive) K_3 may have a significant effect on stabilizing a 3D BEC QB.

Lee-Huang-Yang beyond mean-field correction

Lee-Huang-Yang beyond mean-field correction

- The GP equation is valid in the weak-coupling limit.
- The next order correction(s) for repulsive interction involve higher orders in nonlinearity and can stabilize a quantum ball against collapse.

Lee-Huang-Yang Correction, PR 106, 1135 (1957)

Lee-Huang-Yang found the next order correction of the nonlinear term

$$\mu(n,a) = 4\pi na + 2\pi\alpha n^{3/2} a^{5/2} + \ldots, \qquad \alpha = \frac{64}{3\sqrt{\pi}}, \qquad n = N|\psi|^2$$

in dimensionless unit with $\hbar = m = 1$. As $a \to \infty$, by dimensional argument (unitarity limit)

$$\mu(n,a) \approx \frac{\hbar^2}{2mL^2} = \eta n^{2/3}$$

L is atomic separation, n density, and η a universal constant.

These two results can be combined into the analytic formula:

$$\mu(n,a) = n^{2/3} f(x), \quad f(x) = 4\pi \frac{x + \alpha x^{5/2}}{1 + \frac{\alpha}{2} x^{3/2} + \frac{4\pi\alpha}{\eta} x^{5/2}}, \quad x = a n^{1/3}$$

Beyond-mean-field model: Weak-coupling to unitarity crossover

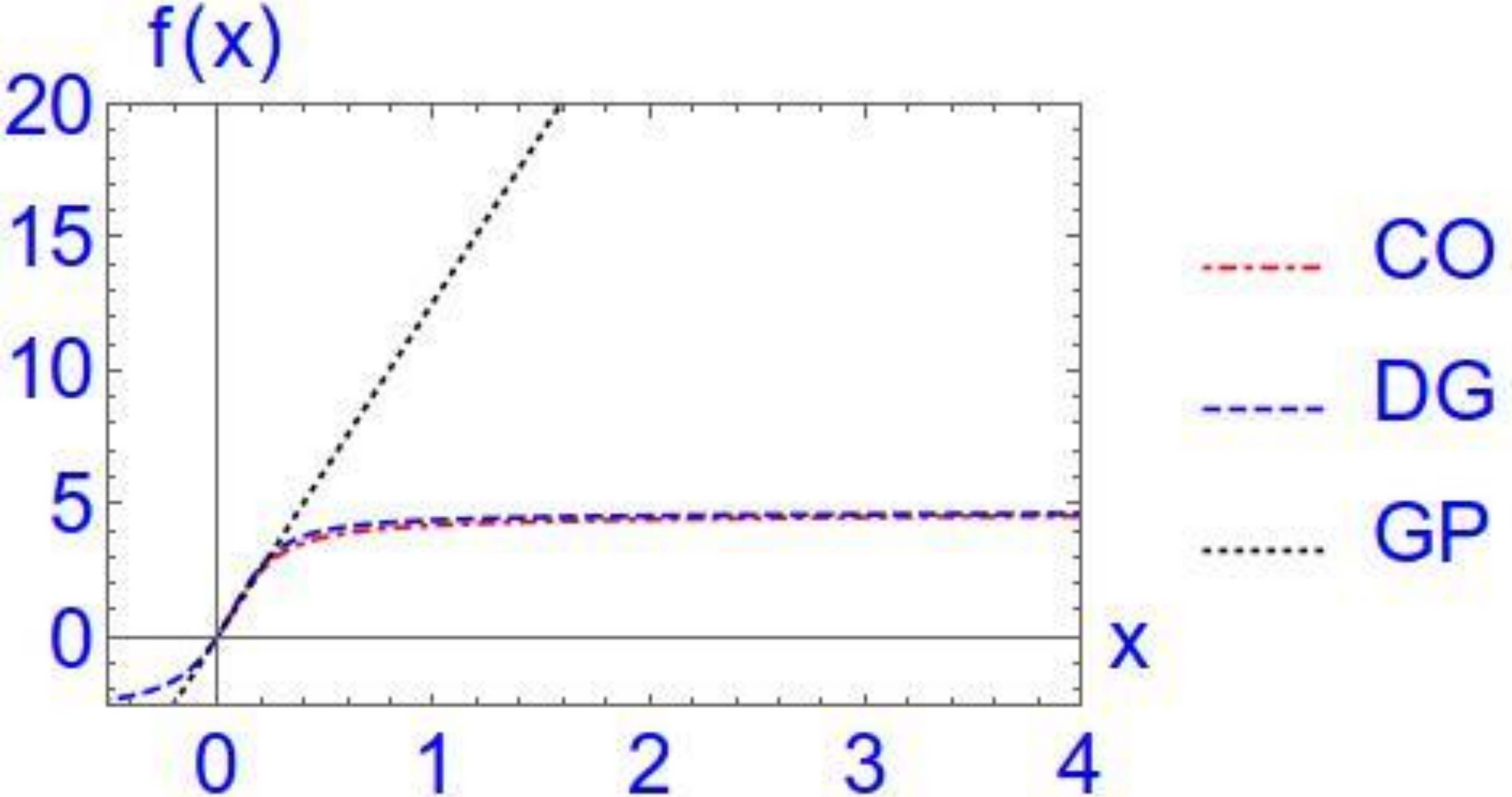
Then we get the modified beyond-mean-field nonlinear Schrödinger equation

$$\left[-\frac{1}{2}\nabla^2 + V(r) + n^{2/3} f(x)\right]\psi(\mathrm{r},t) = i\frac{\partial}{\partial t}\psi(\mathrm{r},t)$$

$$f(x) = 4\pi \frac{x + \alpha x^{5/2}}{1 + \frac{\alpha}{2} x^{3/2} + \frac{4\pi\alpha}{\eta} x^{5/2}}, \quad x = a n^{1/3}, \quad n = N|\psi|^2$$

where $f(x)$ is a universal function. We compare this function with a microscopic Hartree calculation, without Hartree approximation.

SKA+L. Salasnich, unpublished


f(x)
20
15
10
5
0
0
1
2
3
4
x
CO
DG
GP

The three-body interaction and/or LHY correction can stabilize

- A BEC QB
- A dipolar BEC
- A binary BEC QB for attractive interspecies interaction
- A binary boson-fermion QB for attractive boson-fermion interaction

Experiments on QB formation

- H. Kadau, M. Schmitt, M. Wenzel, C. Wink, T. Maier, I. Ferrier-Barbut, and T. Pfau, Nature (London) 530, 194 (2016). → Dipolar BEC with repulsive interaction.
- C. R. Cabrera, L. Tanzi, J. Sanz, B. Naylor, P. Thomas, P. Cheiney, and L. Tarruell, Science 359, 301 (2018) → Quantum liquid droplets in a mixture of Bose-Einstein condensates .
- G. Semeghini, G. Ferioli, L. Masi, C. Mazzinghi, L. Wolswijk, F.Minardi, M. Modugno, G. Modugno, M. Inguscio, and M. Fattori, arXiv:1710.10890 → Self-bound quantum droplets in atomic mixtures

Generalized Gross-Pitaevskii (GP) Equation (mean-field equation for the BEC)

$$i\hbar \frac{\partial \psi(\mathrm{r},t)}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 - \frac{4\pi\hbar^2 |a| N}{m}|\psi|^2 + \frac{\hbar N^2 K_3}{2}|\psi|^4 \right]\psi(\mathrm{r},t);$$

Dynamics

$$= \mu\psi(\mathrm{r},t);$$

Stationary state

Dimensionless GP equation

$$i\frac{\partial \psi(\mathrm{r},t)}{\partial t} = \left[-\frac{1}{2}\nabla^2 - 4\pi |a| N|\psi|^2 + \frac{N^2 K_3}{2} |\psi|^4 \right]\psi(\mathrm{r},t)$$

Unit of length $l_0 = 1\,\mu\mathrm{m}$

Unit of time $t_0 = \dfrac{m l_0{}^2}{\hbar} = 0.11\,\mathrm{ms}$

Unit of energy $\varepsilon_0 = \dfrac{\hbar^2}{m l_0{}^2} \approx 10^{-30}$ J

Unit of $K_3 = \dfrac{\hbar l^4}{m}$

Results reported are for ^{7}Li atoms, e.g.,

$$a = -27.4 a_0, \quad \mathrm{m} = 7\,\mathrm{amu}.$$

All results will be expressed in actual physical units.

Variational Approximation

Variational Gaussian Ansatz for wavefunction

$$\psi(\mathrm{r}) = \frac{\pi^{-3/4}}{w^{3/2}} \exp\left[-\frac{r^2}{2w^2}\right], \quad w -> \text{width of the QB}$$

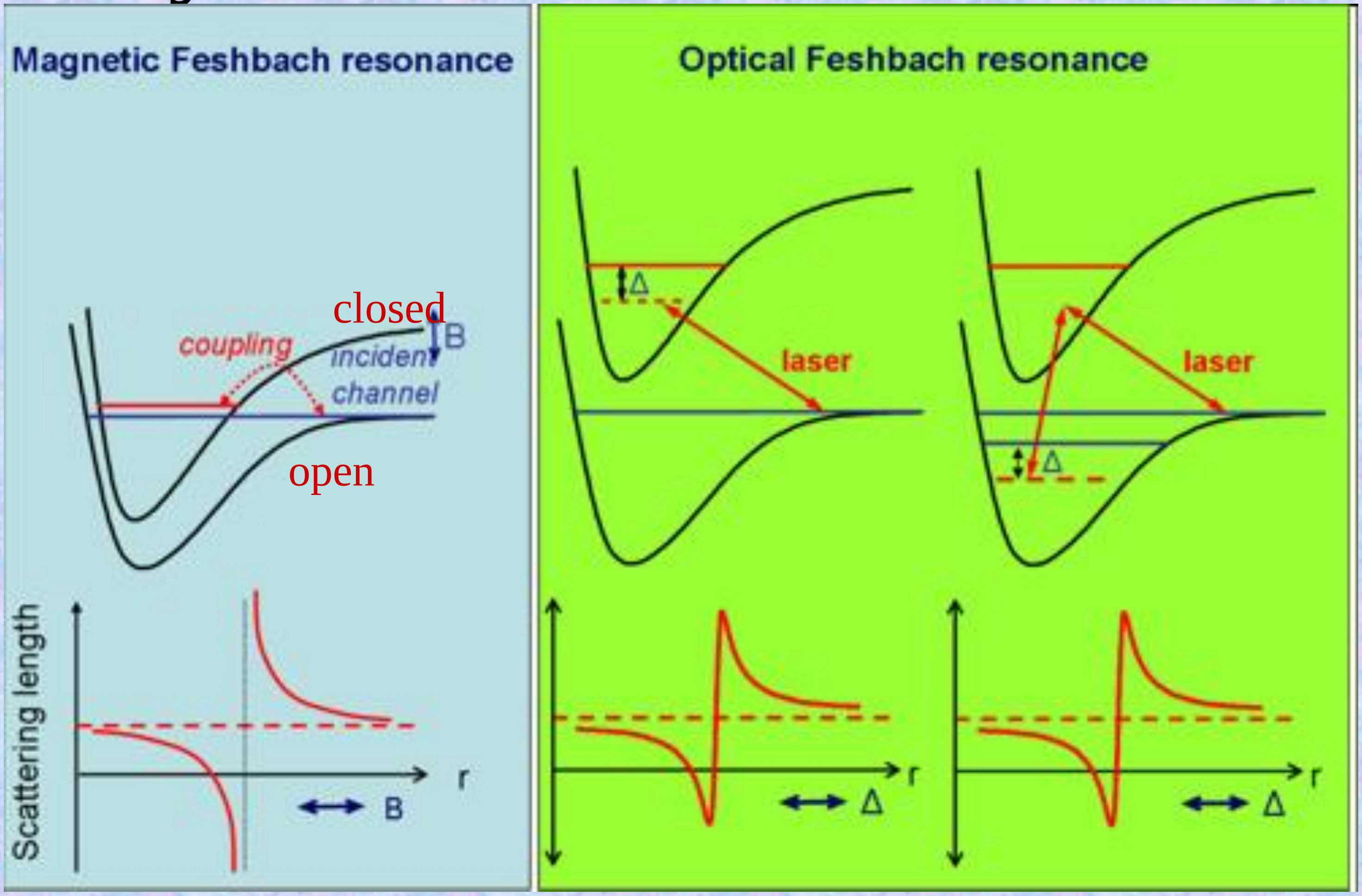
$$\varepsilon(\mathrm{r}) = \frac{|\nabla\psi(\mathrm{r})|^2}{2} - 2\pi N|a||\psi(\mathrm{r})|^4 + \frac{K_3 N^2}{6}|\psi(\mathrm{r})|^6,$$

$$E = \int \varepsilon(\mathrm{r})d\mathrm{r} = \frac{3}{4w^2} - 2\pi N|a|\frac{\pi^{-3/2}}{2\sqrt{2}w^3} + \frac{K_3 N^2}{2}\frac{\pi^{-3}}{9\sqrt{3}w^6},$$

Energy minimum determines width w:

$$\frac{dE}{dw} = 0, --> \qquad \frac{1}{w^3} - \frac{4\pi N|a|}{(2\pi)^{3/2}w^4} + \frac{N^2 K_3}{2}\frac{4}{9\sqrt{3}\pi^3 w^7} = 0$$

Tuning of short-range interaction by a Feshbach resonance



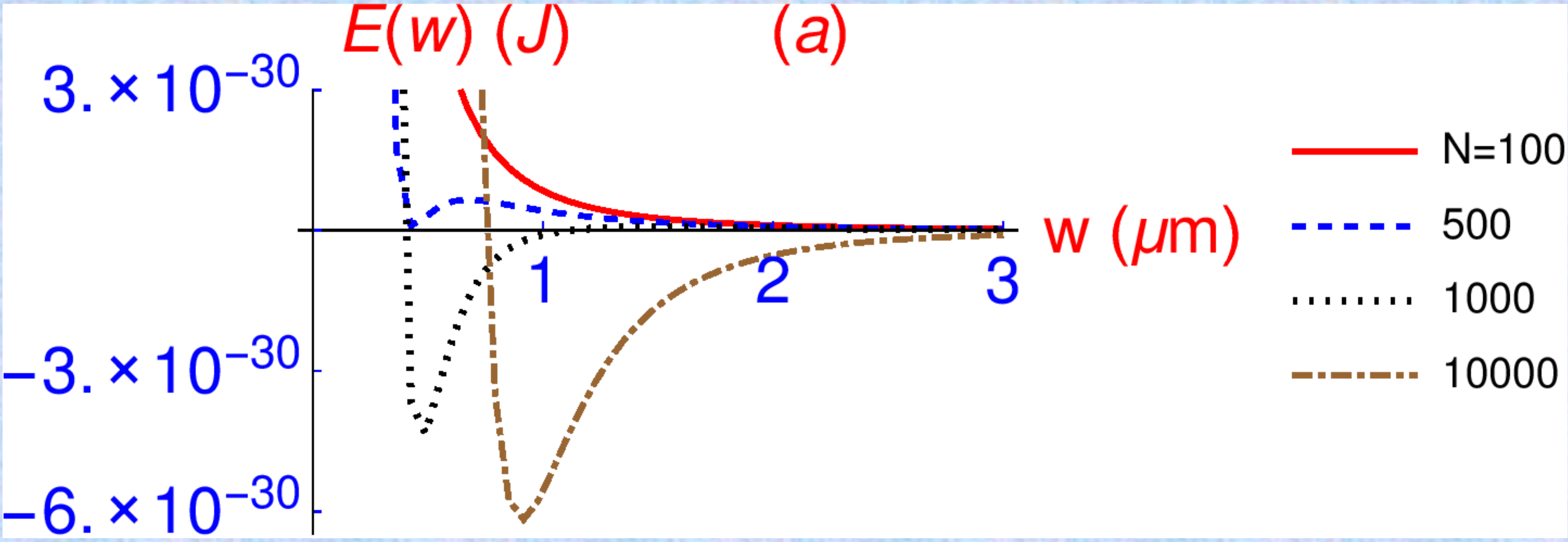
Binding mechanism of a 3D QB

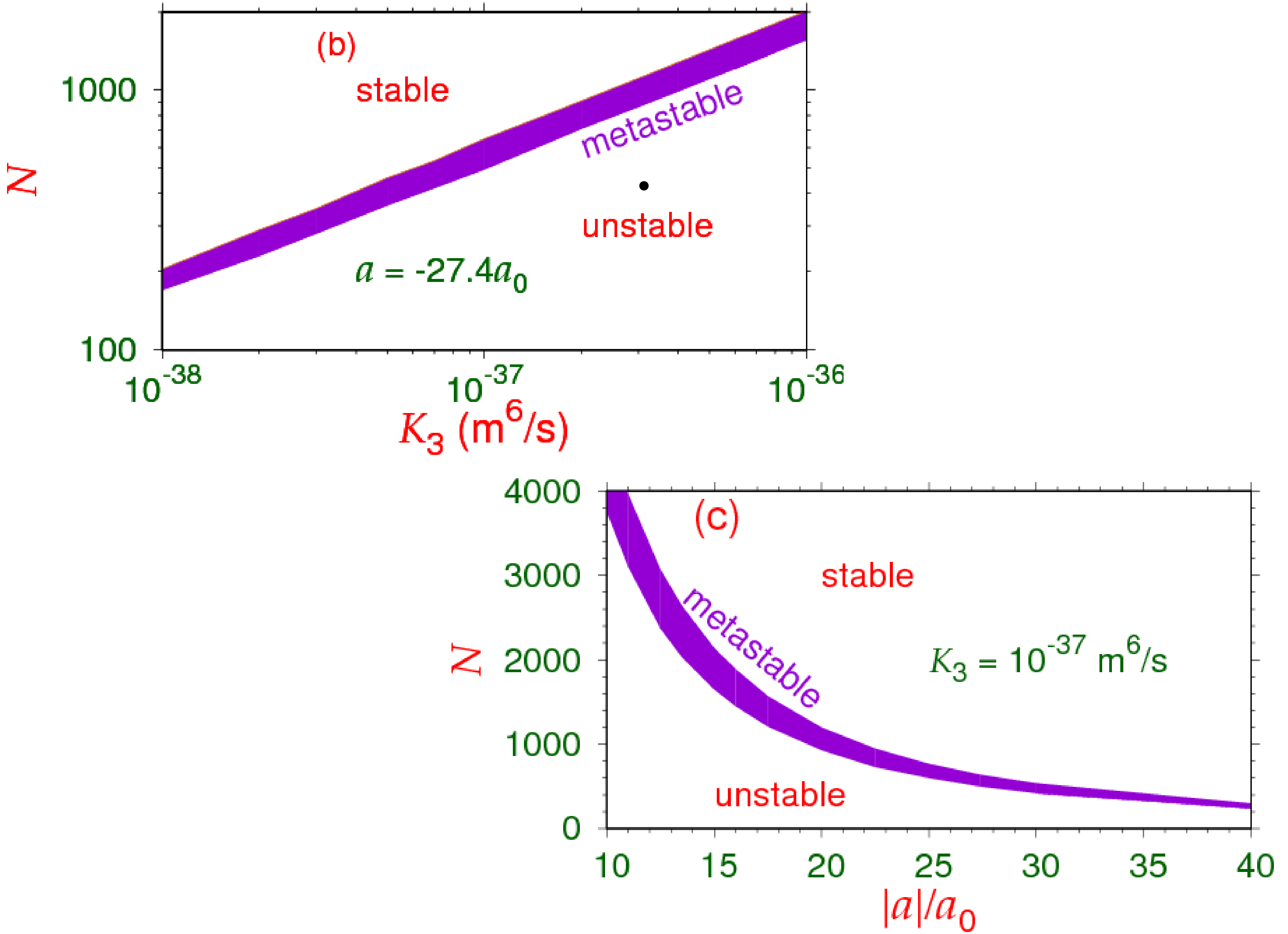
- Problems in QB binding:

  weak attraction -> leakage & strong attraction ->collapse
- Attraction provided by Feshbach technique to manipulate scattering length to a negative value
- Collapse stopped by a small three-body repulsion K_3. The energy is + infinity at the centre. This eliminates the possibility of a collapsed state.

Variational energy vs width of a QB

Stable and metastable states




(b)
stable
metastable
unstable
a = -27.4a_0
1000
100
N
10^{-38}
10^{-37}
10^{-36}
K_3 (m^6/s)
(c)
stable
metastable
unstable
K_3 = 10^{-37} m^6/s
4000
3000
2000
1000
0
N
10
15
20
25
30
35
40
$|a|/a_0$

Real-time propagation

Numerical Solution :

Real - time propagation :

$$i\frac{\partial\psi(x,t)}{\partial t}=\left[-\frac{1}{2}\frac{\partial^2}{\partial x^2}+\frac{1}{2}x^2+g\,|\psi|^2\right]\psi(x,t)$$

Stationary state : Start with a solution of the linear equation ($g=0$). Propagate this solution for a small time dt setting $g = dg$. As $dt \to 0$, $dg \to 0$ this yields the solution of the GP equation with $g = dg$. Repeat this iteration many times so that the solution of the GP equation with a finite g is obtained.

Non - stationary state : It is also possible to study dynamics.

Imaginary-time propagation

Numerical Solution :

Imaginary-time propagation for stationary ground state :

Set $t = -i\tau$,

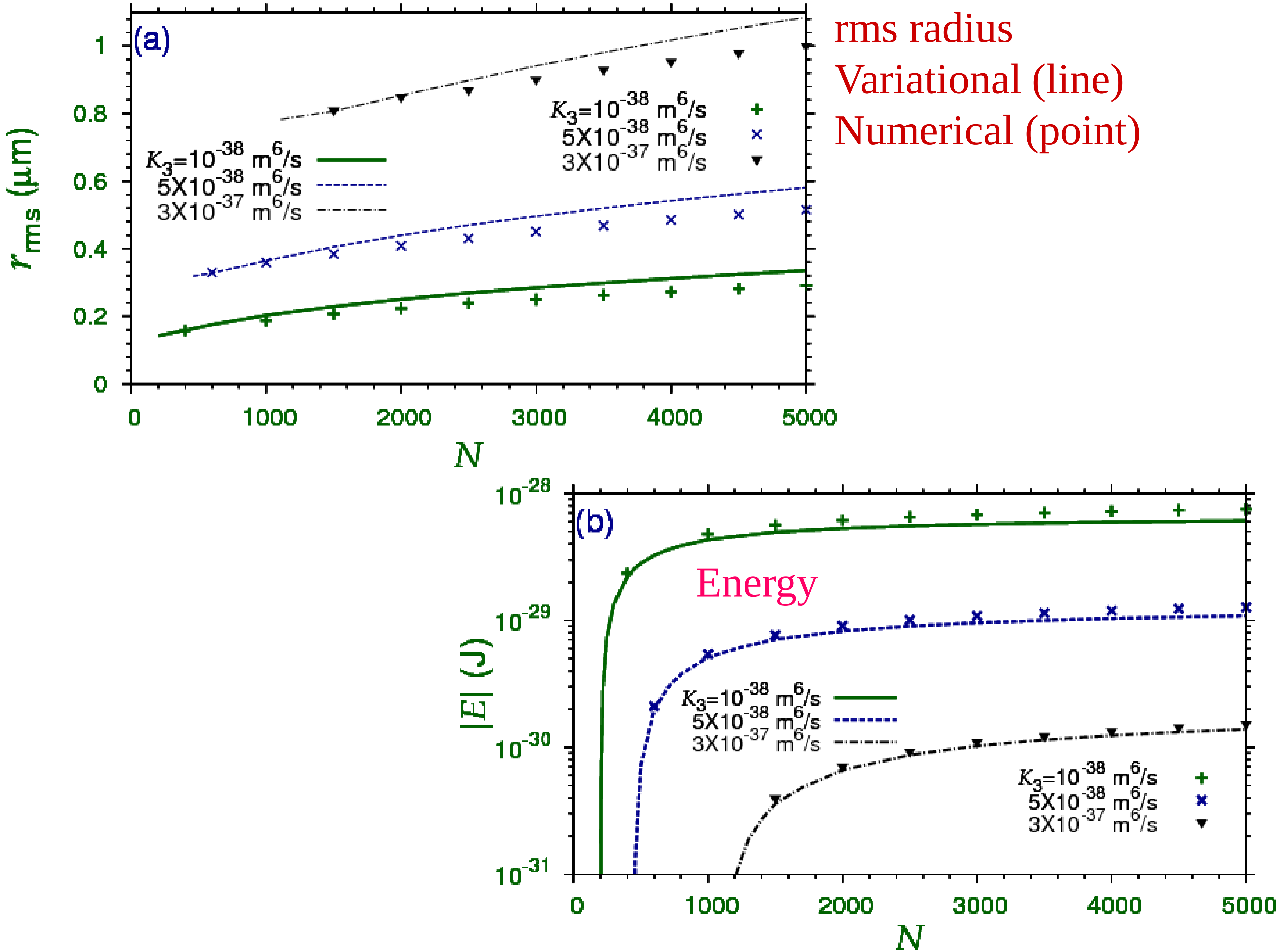
$$-\frac{\partial \psi(x,\tau)}{\partial \tau} = \left[-\frac{1}{2}\frac{\partial^2}{\partial x^2} + \frac{1}{2}x^2 + g\,|\psi|^2 \right]\psi(x,\tau) \equiv E_0\psi(x,\tau)$$

Time propagation yields $\psi(x,\tau) = \psi(x,0)\exp(-E_0\tau)$.

An arbitrary initial state $\psi(x,0)$ is usually a linear combination of all eigenstates n. As a result of time propagation all states decay. The excited states n decay much rapidly as they have larger energy $(E_n > E_0)$. So after some time only the ground state remains. Involves real algebra only.

Numerical Solution

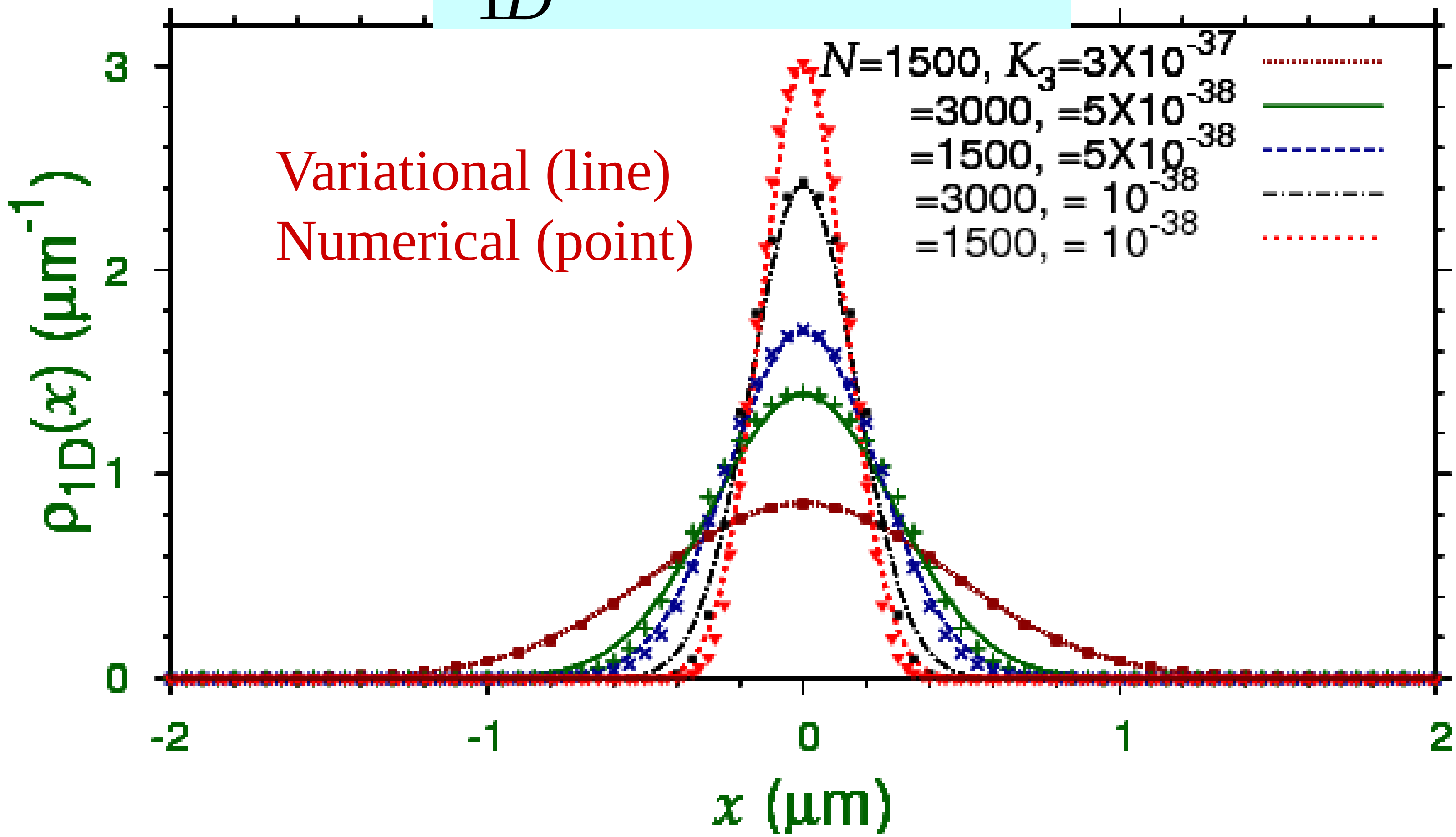
- We solve the 3D GP equation by split-step Crank-Nicolson method using both real- and imaginary-time propagation in Cartesian coordinates employing a space step 0.025 and a time step 0.0002 using Fortran and C programs published by us in Comput. Phys. Commun.



rms radius
Variational (line)
Numerical (point)
(a)
r_{rms} (μm)
N
K_3=10^-38 m^6/s
5X10^-38 m^6/s
3X10^-37 m^6/s
(b)
Energy
|E| (J)
N
K_3=10^-38 m^6/s
5X10^-38 m^6/s
3X10^-37 m^6/s

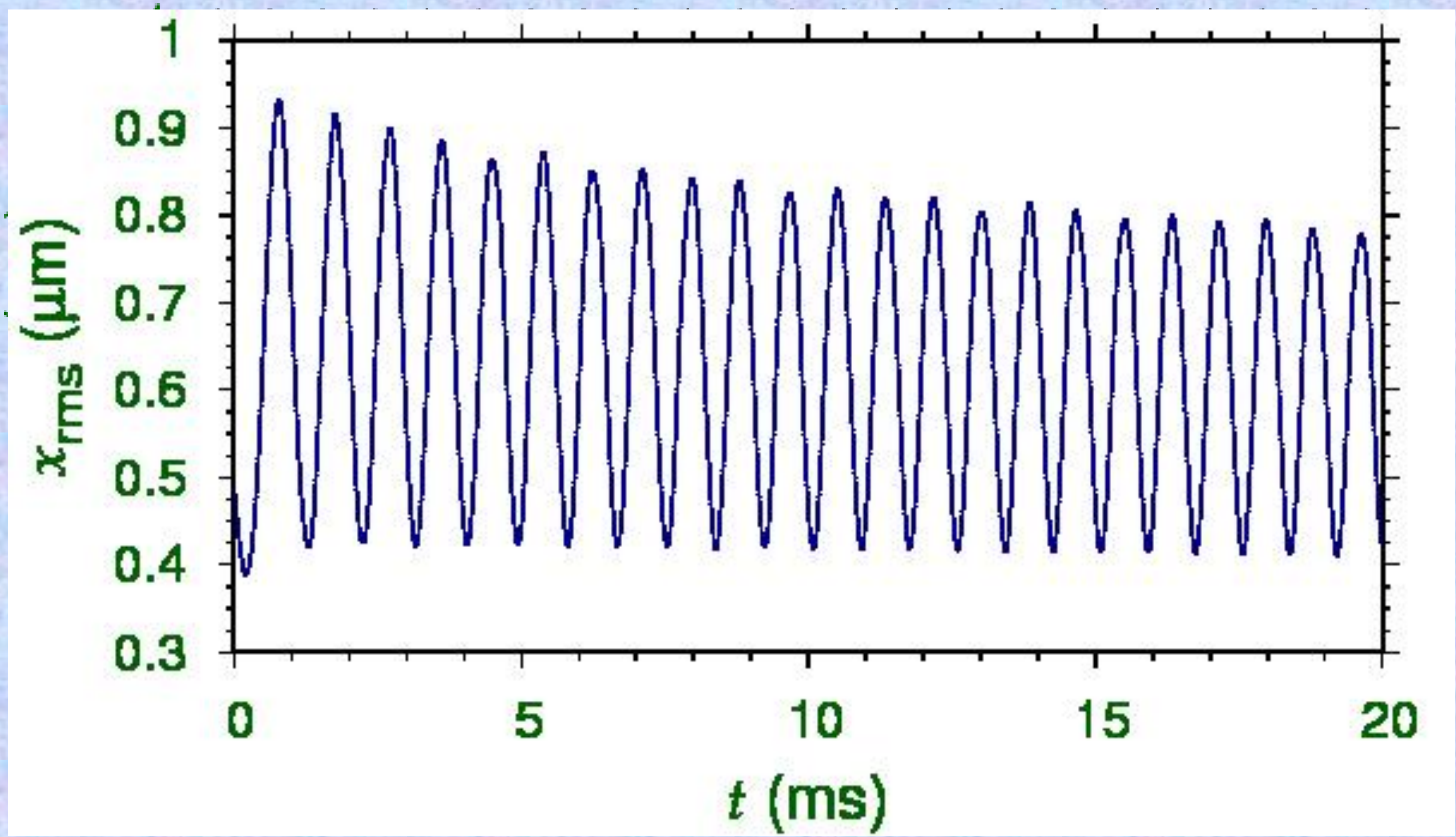
Variational (line) and numerical (point) one-dimensional (1D) density

$$\rho_{1D}(x) = \int |\psi(\mathrm{r})|^2 \, dydz$$



Dynamics of a N=1500, K_3=3X10^{-37} m^6/s QB at t = 0 changed to

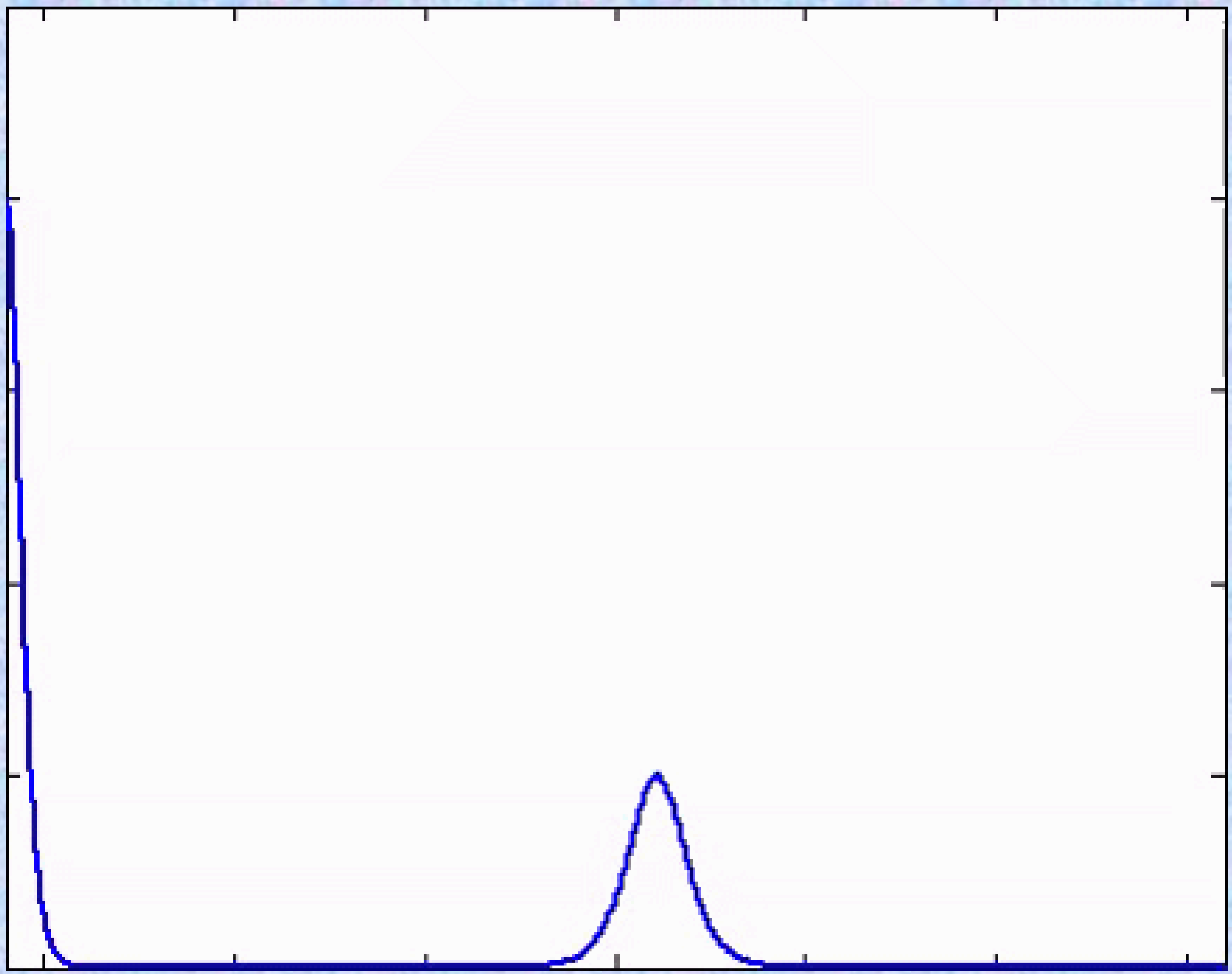
K_3=3(1-0.1i)X10^{-37} m^6/s



Soliton in one dimension (1D)

- A 1D **soliton** is a solitary wave that maintains its shape while travelling.
- It is generated from a balance between repulsive kinetic energy and attractive nonlinear interaction.
- Analytic soliton: Energy momentum conservation
- Elastic collision: Two 1D solitons can pass through each other in collision without a change in shape.

Soliton-soliton collision



Set the QBs in motion

Multiply stationary wavefunction by $\exp(ipx/\hbar)$

$$\psi(\mathrm{r}) \Rightarrow \psi(\mathrm{r})\exp(ipx/\hbar),$$

where momentum $p = mv$

with v the generated velocity

Numerically this is achieved by accurate (real-time) simulation with small space and time steps over a large domain of space

Quasi Soliton in 3D

- No rigorous energy momentum conservation
- Inelastic collision: Solitonic nature is approximate and there is some change of shape of quantum balls during collision.

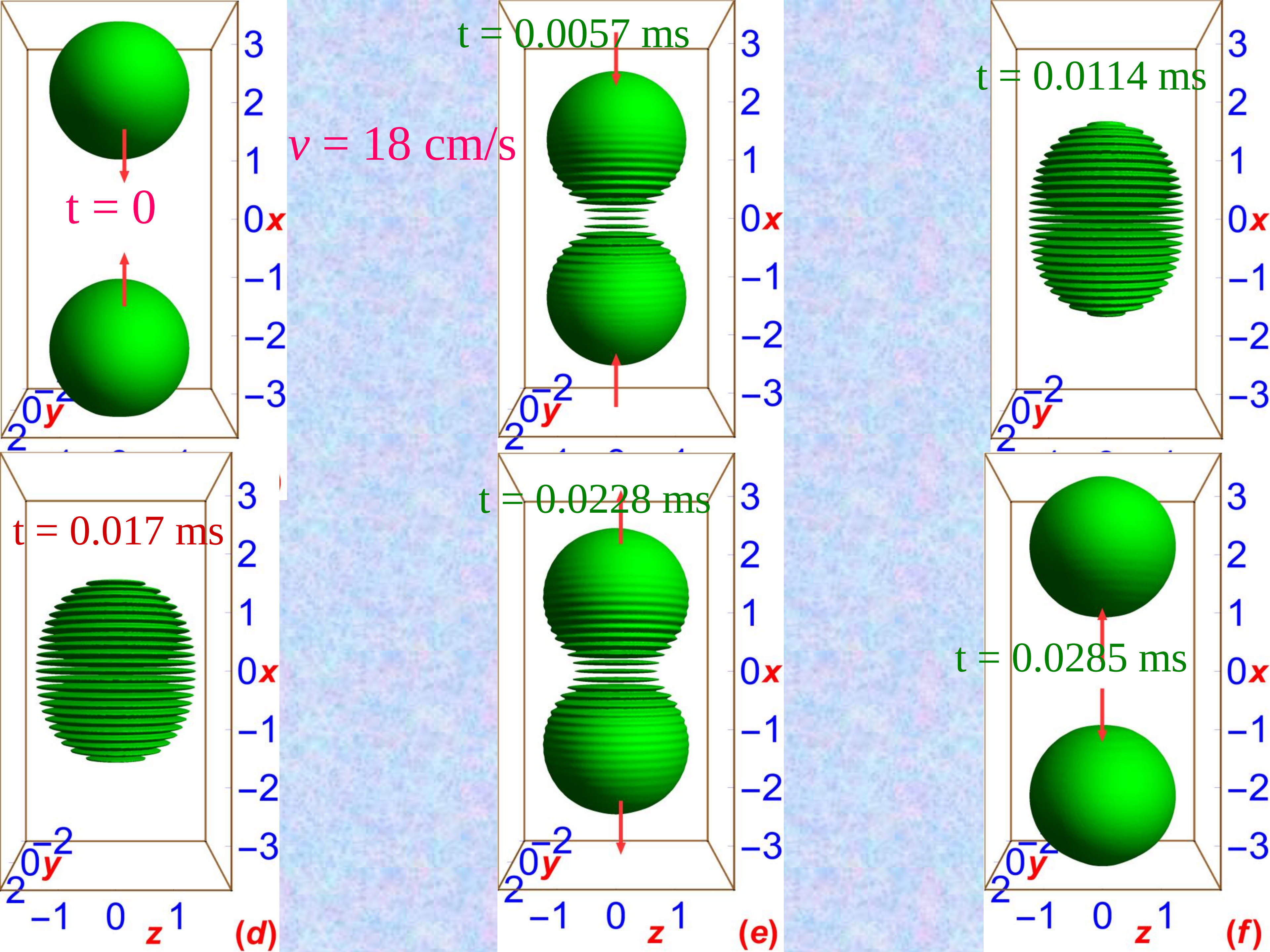
Collision of 2 QBs: 3 types of collision

- Large velocities: Elastic collision (small encounter time, kinetic energy of motion much larger than internal energies)
  Frontal and angular collision and that with an impact parameter
- Small velocities: Inelastic collision and destruction of QBs, formation of bound QBs (large encounter time, kinetic energy of motion much smaller than internal energies)
- Intermediate velocities: Collision with some change in shape.

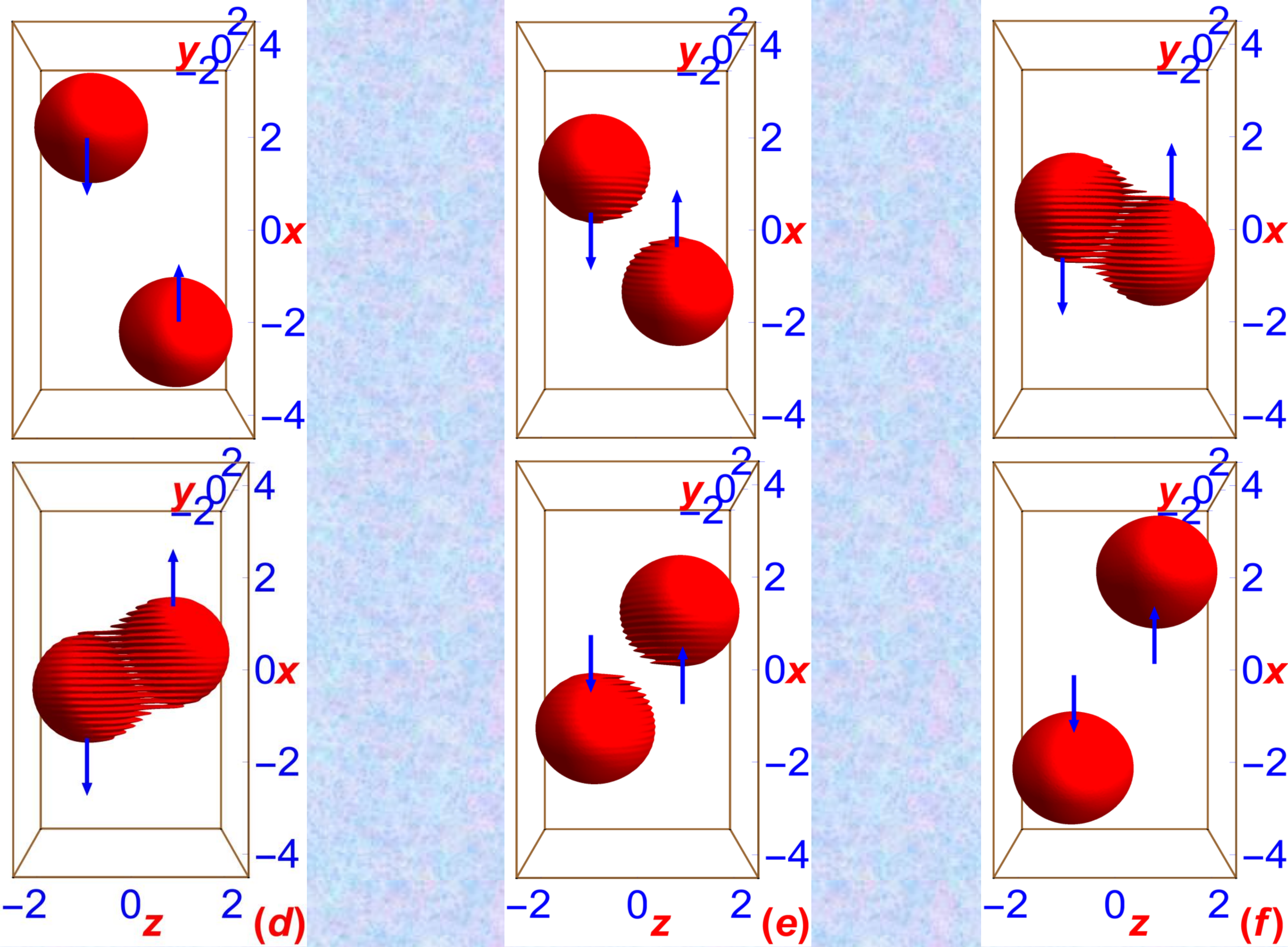
Collision of two ^{7}Li balls, with N = 1500,

$K_3 = 3X10^{-37}(1-i)\ m^6/s$

Moving in opposite directions along x axis with velocity 18 cm/s, at times
(a) t = 0, (b) = 0.0057 ms, (c) = 0.0114 ms, (d) = 0.017 m
(e) = 0.0228 ms, (f) = 0.0285 ms. The density on the con
is 10^{10} atoms/cm^3 and unit of length is μm.

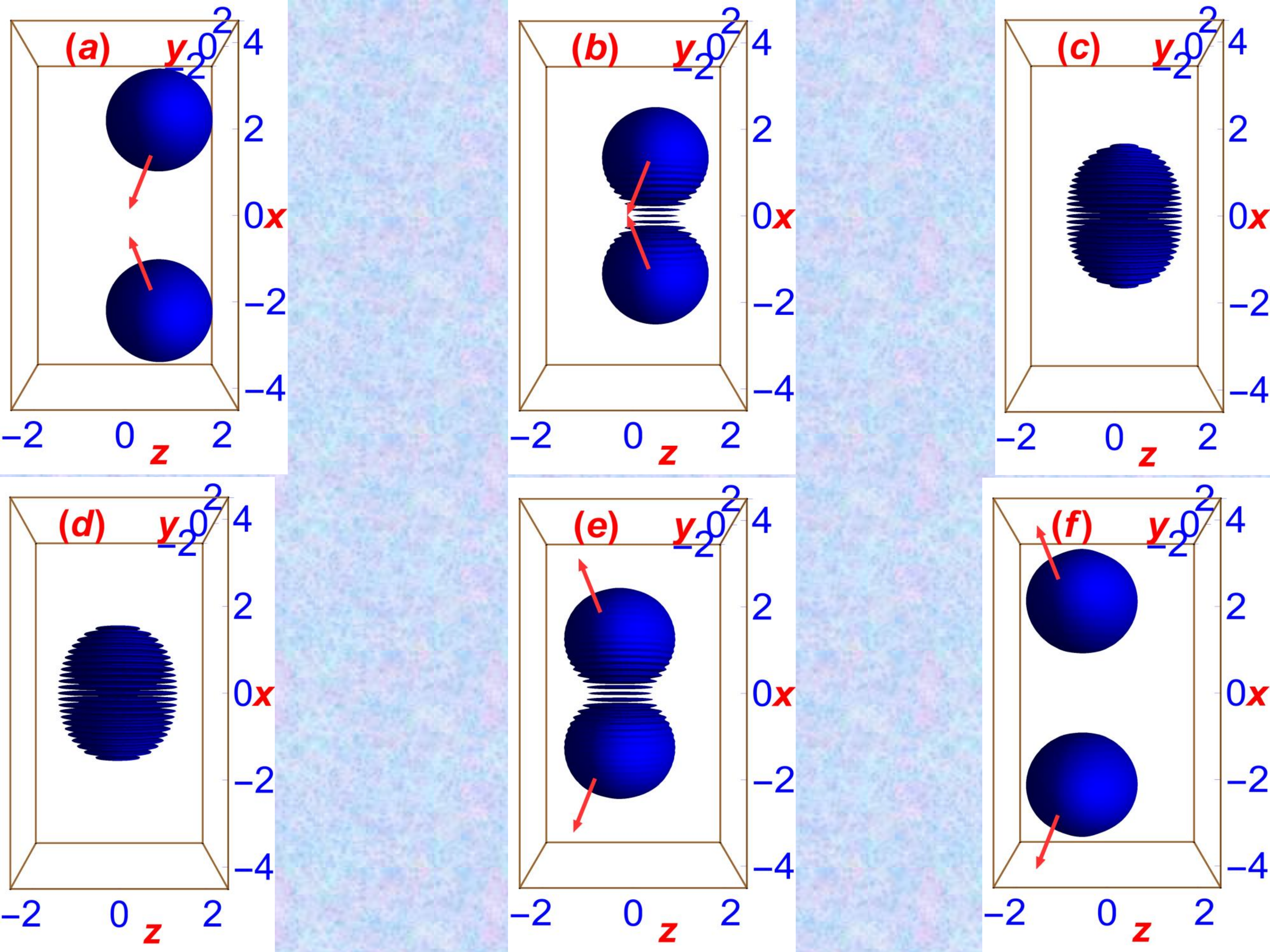


t = 0
v = 18 cm/s
t = 0.0057 ms
t = 0.0114 ms
t = 0.017 ms
t = 0.0228 ms
t = 0.0285 ms
3
2
1
0 x
−1
−2
−3
y
−1 0 z 1
(d)
(e)
(f)

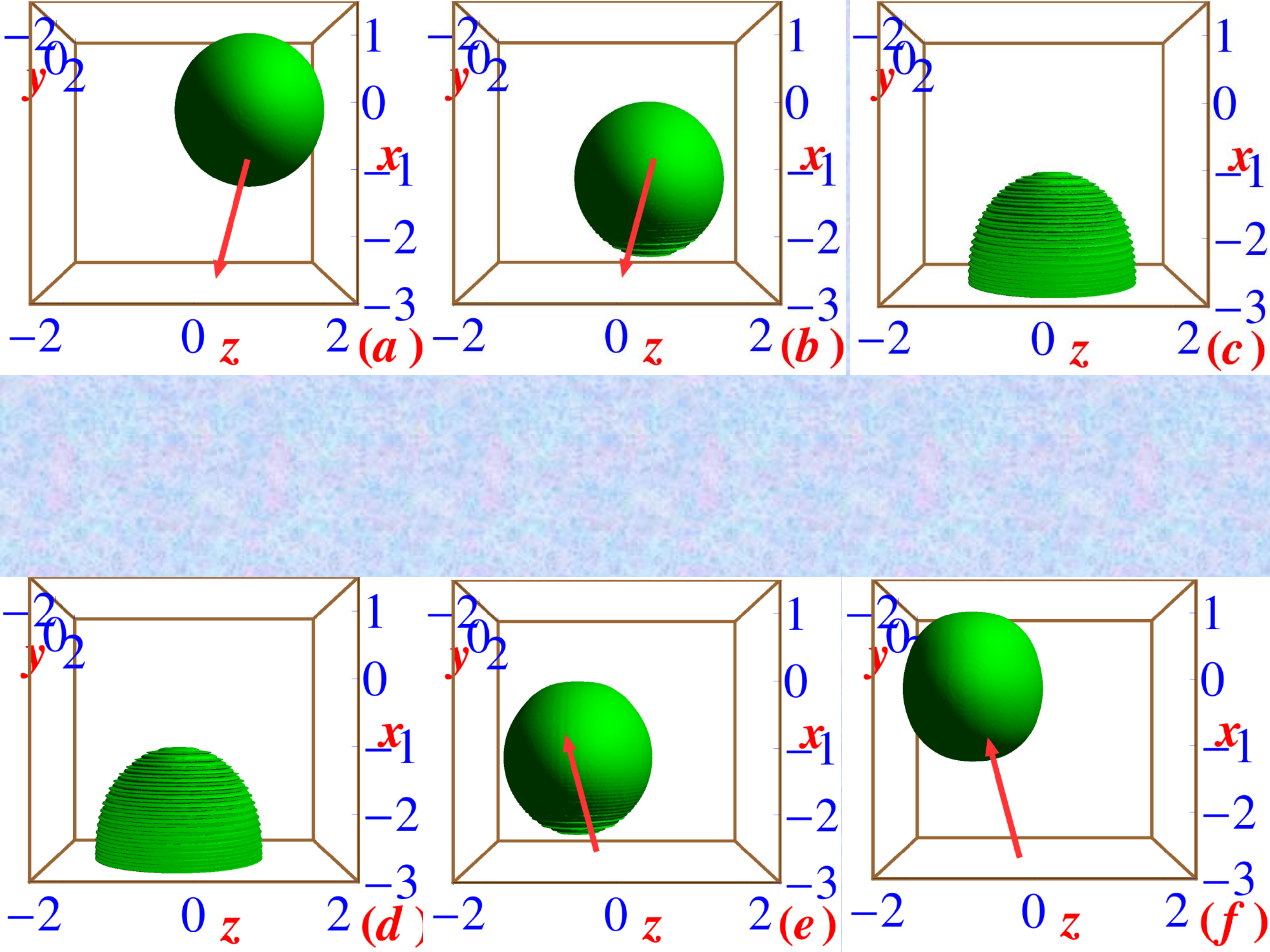

y
x
z
−2
0
2
4
−4
(d)
(e)
(f)

Angular Collision of two ^7Li balls, with N = 1500,
$K_3 = 3X10^{-37}(1-i)\ m^6/s$

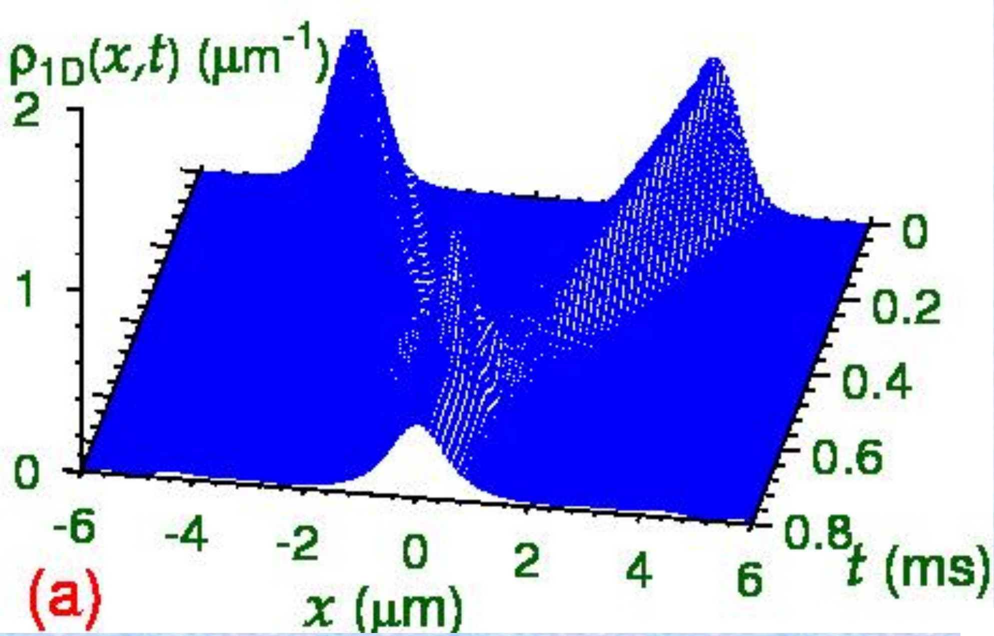
Angular collision with velocity 19 cm/s, at times
(a) t = 0, (b) = 0.0057 ms, (c) = 0.0114 ms, (d) = 0.017 m
(e) = 0.0228 ms, (f) = 0.0285 ms. The density on the con
is 10^{10} atoms/cm^3 and unit of length is μm.


(a)
(b)
(c)
(d)
(e)
(f)
x
y
z
4
2
0
−2
−4

Bouncing of a ^{7}Li ball, with N = 1500, $K_3 = 3X10^{-37}(1-i)\ m^6/s$, against a hard rigid noninteracting wall

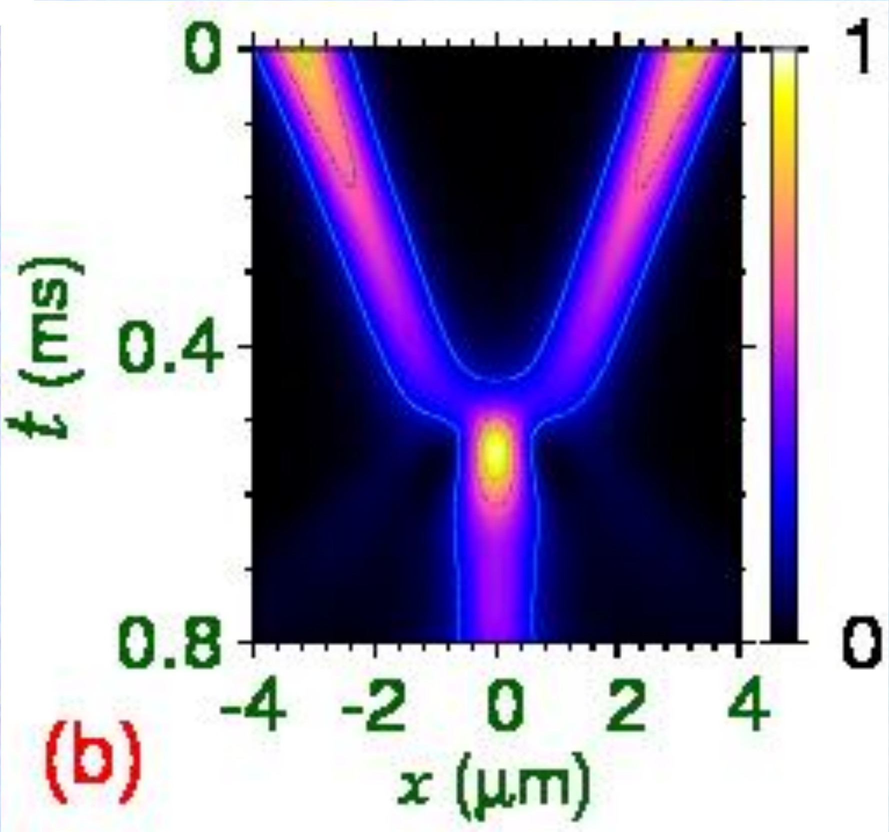


(a)
(b)
(c)
(d)
(e)
(f)
x
y
z
−2
0
2
1
0
−1
−2
−3

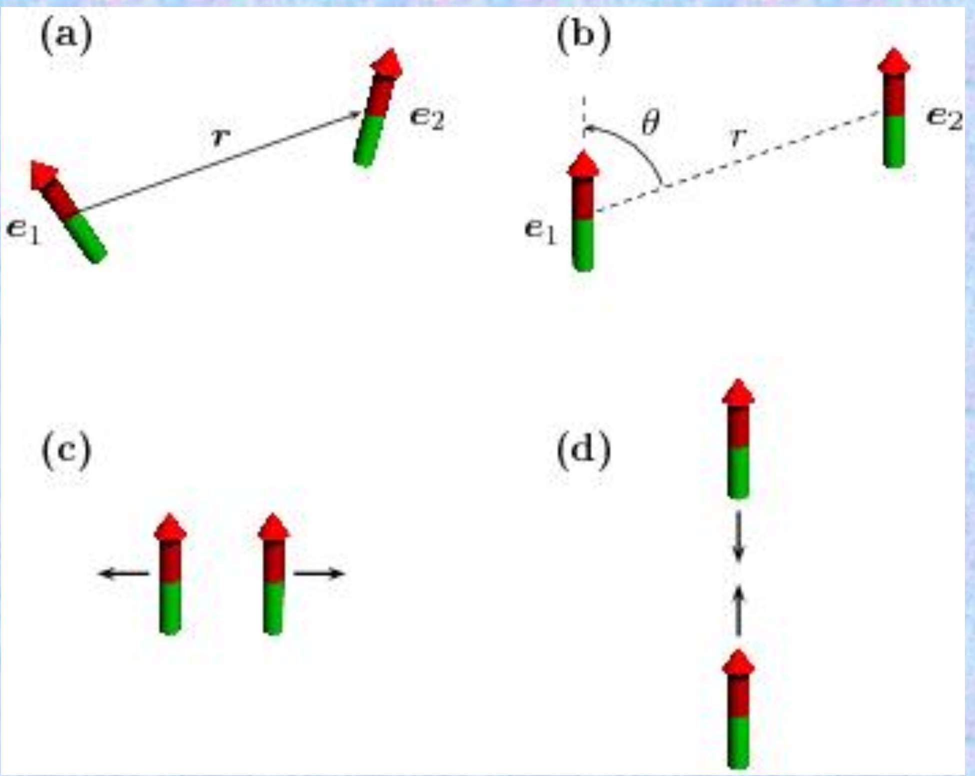


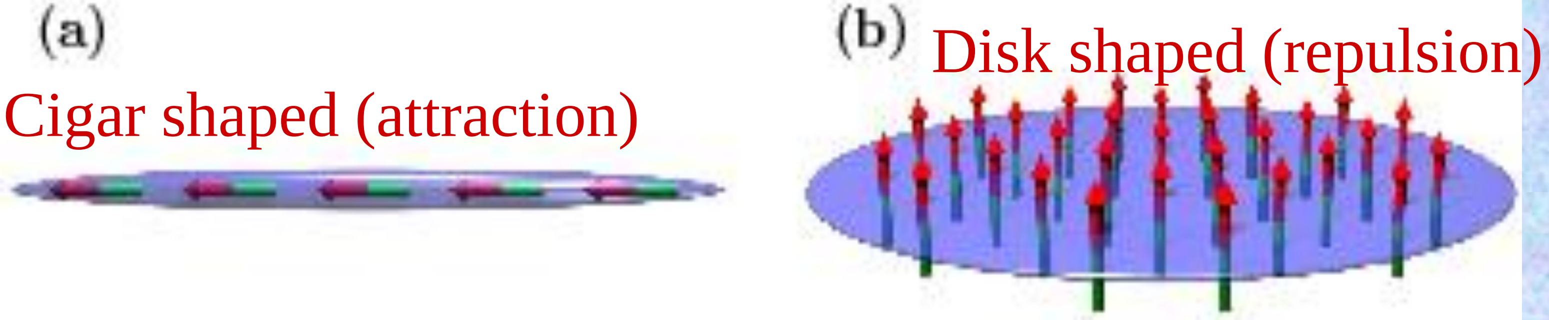
Molecule formation at small velocities

Quantum balls with
N = 1500. $K_3 = 3X10^{-37}(1\text{-}0.05i)\ m^6/s$
placed at x = 3.2/-3.2 micron at t = 0
and moving with velocity 0.45 cm/s



Dipolar interaction: Atoms and molecules





Static Dipole-Dipole Interactions

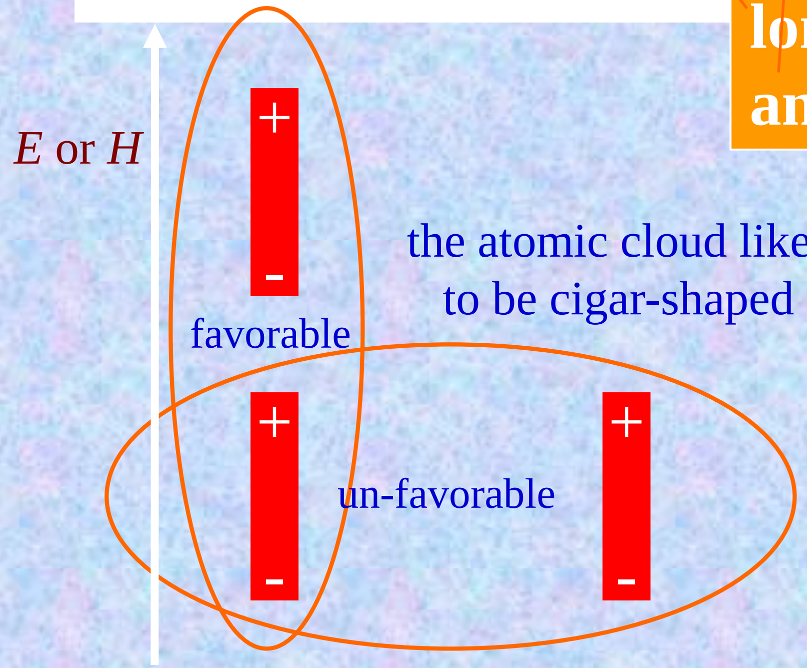
Magnetic dipole-dipole interaction: the magnetic moments of the atoms are aligned with a strong magnetic field [Goral, Rzazewski, and Pfau, 2000]

$$U_{dd}(r) = \frac{\mu_0 \mu^2}{4\pi}\left[\frac{1-3\cos^2\theta}{r^3}\right]$$

Electrostatic dipole-dipole interaction: (i) permanent electric moments (polar molecules); (ii) electric moments induced by a strong electric field E [Yi and You 2000; Santos, Shlyapnikov, Zoller and Lewenstein 2000]

$$U_{dd}(r) = \frac{\alpha^2}{4\pi\varepsilon_0}\left[\frac{1-3\cos^2\theta}{r^3}\right]$$

long-range + anisotropic



the atomic cloud likes to be cigar-shaped

$$a_{dd} = \frac{\mu_0 \mu^2 m}{12\pi\hbar^2}$$

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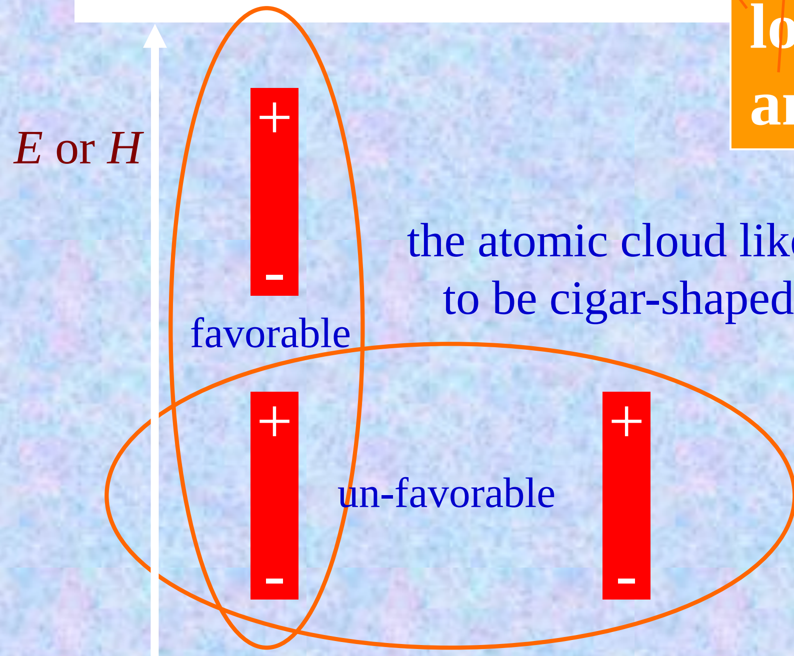
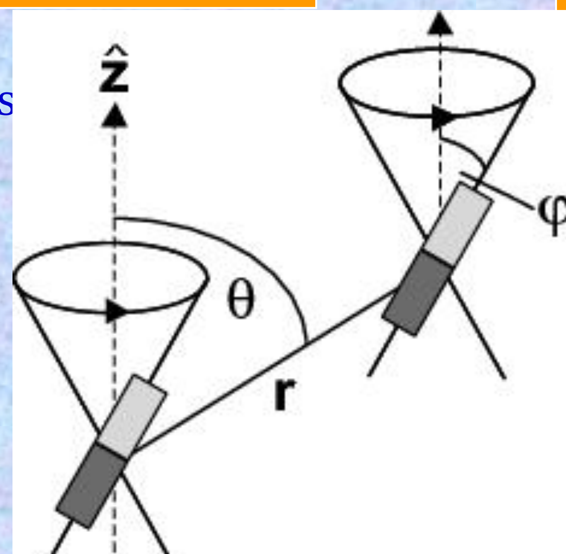
$$U_{dd}(r) = \frac{\alpha^2}{4\pi\varepsilon_0}\left[\frac{1-3\cos^2\theta}{r^3}\right]$$

long-range + anisotropic

tunability

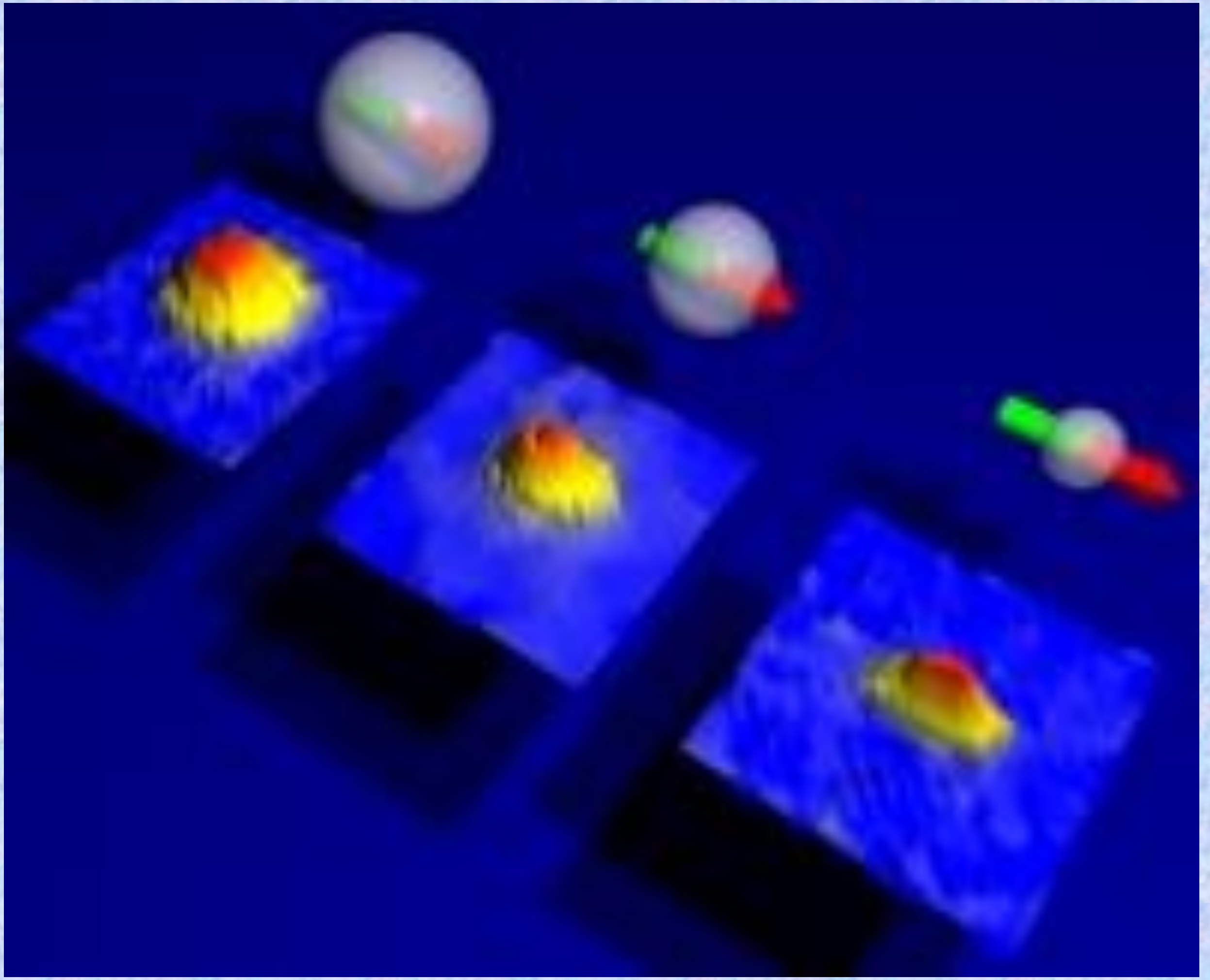


the atomic cloud likes to be cigar-shaped



$$a_{dd} = \frac{\mu_0 \mu^2 m}{12\pi\hbar^2}$$

Change of shape of BEC as the atomic interaction is reduced in a dipolar BEC



BECs of ^{52}Cr (Griesmaier/Pfau 2005), ^{164}Dy (Lu/Lev 2011), ^{168}Er (Ferlaino 2012)

Dipole moment μ of $^{52}Cr = 6\mu_{B,}$ $a_{dd} = 15.3\ a_0$

Dipole moment μ of $^{168}Er = 7\mu_B$ $a_{dd} = 66.7\ a_0$

Dipole moment μ of $^{164}Dy = 10\mu_B$ $a_{dd} = 132.7\ a_0$

Dipole moment μ of $^{87}Rb = 1\mu_B$ $a_{dd} = 0.69\ a_0$

μ_B = Bohr Magneton

a_0 = Bohr radius

$$a_{dd} = \frac{\mu_0 \mu^2 m}{12\pi\hbar^2}$$

Generalized Gross-Pitaevskii (GP) Equation (mean-field equation for the BEC)

$$i\hbar\frac{\partial\psi(\mathrm{r},t)}{\partial t}=-\left[\frac{\hbar^2}{2m}\nabla^2-\frac{4\pi\hbar^2 aN}{m}|\psi|^2-\frac{\hbar N^2K_3}{2}|\psi|^4\right]\psi(\mathrm{r},t)$$

$$+\frac{3\hbar^2 a_{dd}N}{m}\int U_{dd}(\mathrm{r}-\mathrm{r}')|\psi(\mathrm{r}',t)|^2 d\mathrm{r}'\psi(\mathrm{r}',t)$$

$$+\frac{2\hbar^2}{m}\alpha\pi a^{5/2}N^{3/2}|\psi|^3\psi;$$

$$a_{dd}=\frac{\mu_0\mu^2 m}{12\pi\hbar^2}$$

Dynamics

$$=\mu\psi(\mathrm{r},t);$$

Stationary state

Dimensionless Gross-Pitaevskii (GP) Equation for attractive interaction

$$i\frac{\partial\psi(\mathrm{r},t)}{\partial t} = -\left[\frac{1}{2}\nabla^2 + 4\pi\,|a|\,N|\psi|^2 - \frac{N^2K_3}{2}|\psi|^4\right]\psi(\mathrm{r},t)$$

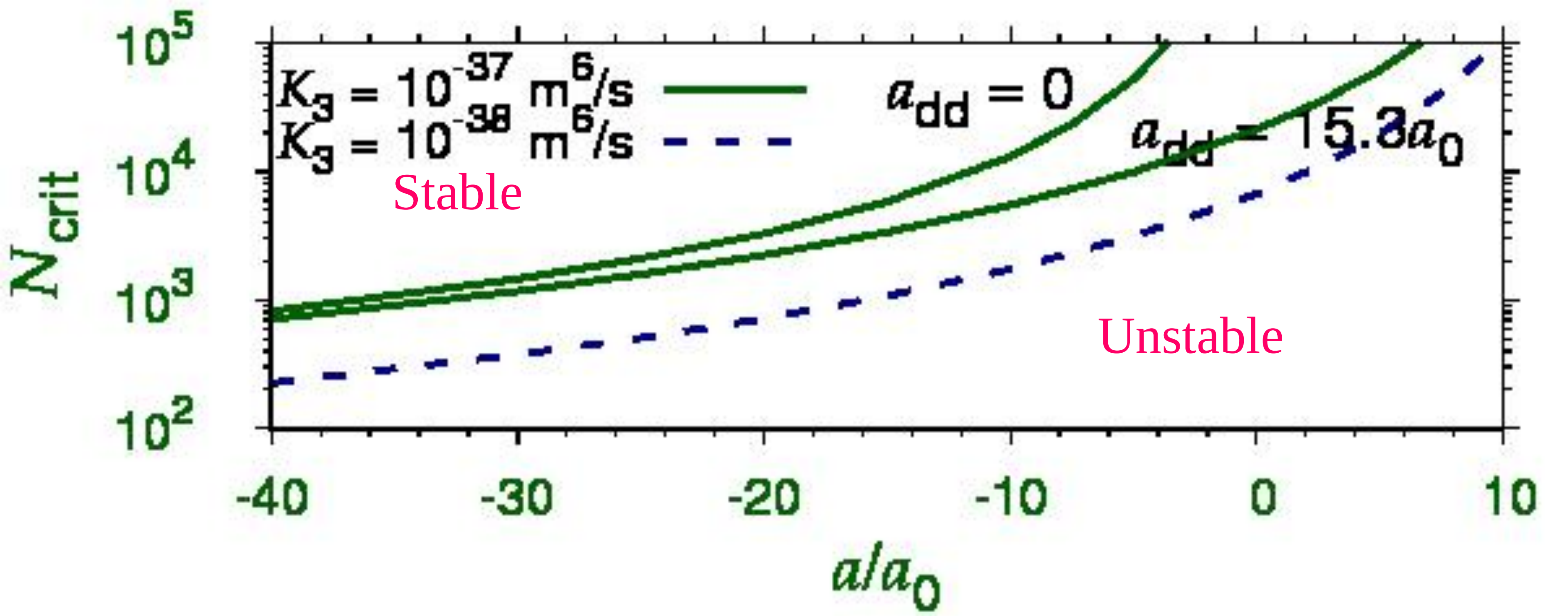
$$+3a_{dd}N\int U_{dd}(\mathrm{r}-\mathrm{r}')|\psi(\mathrm{r}',t)|^2 d\mathrm{r}'\psi(\mathrm{r}',t);$$ Dynamics

$$= \mu\psi(\mathrm{r},t);$$ Stationary state

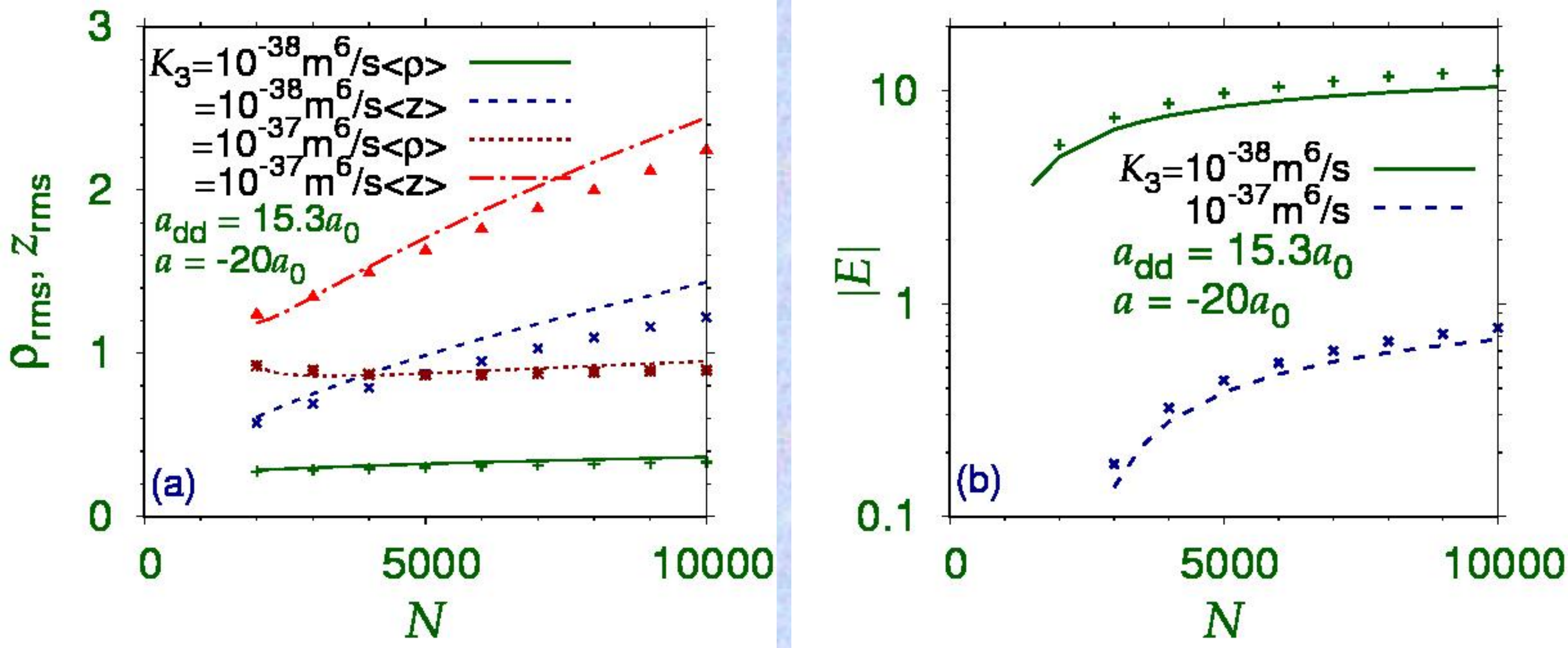
Parameters:

- Work with ^{52}Cr atom with magnetic moment $\mu = 6\,\mu_B$
- $a = -20a_0$, $a_{dd}=15.3a_0$
- Unit of length 1μm
- Unit of time 0.82 ms

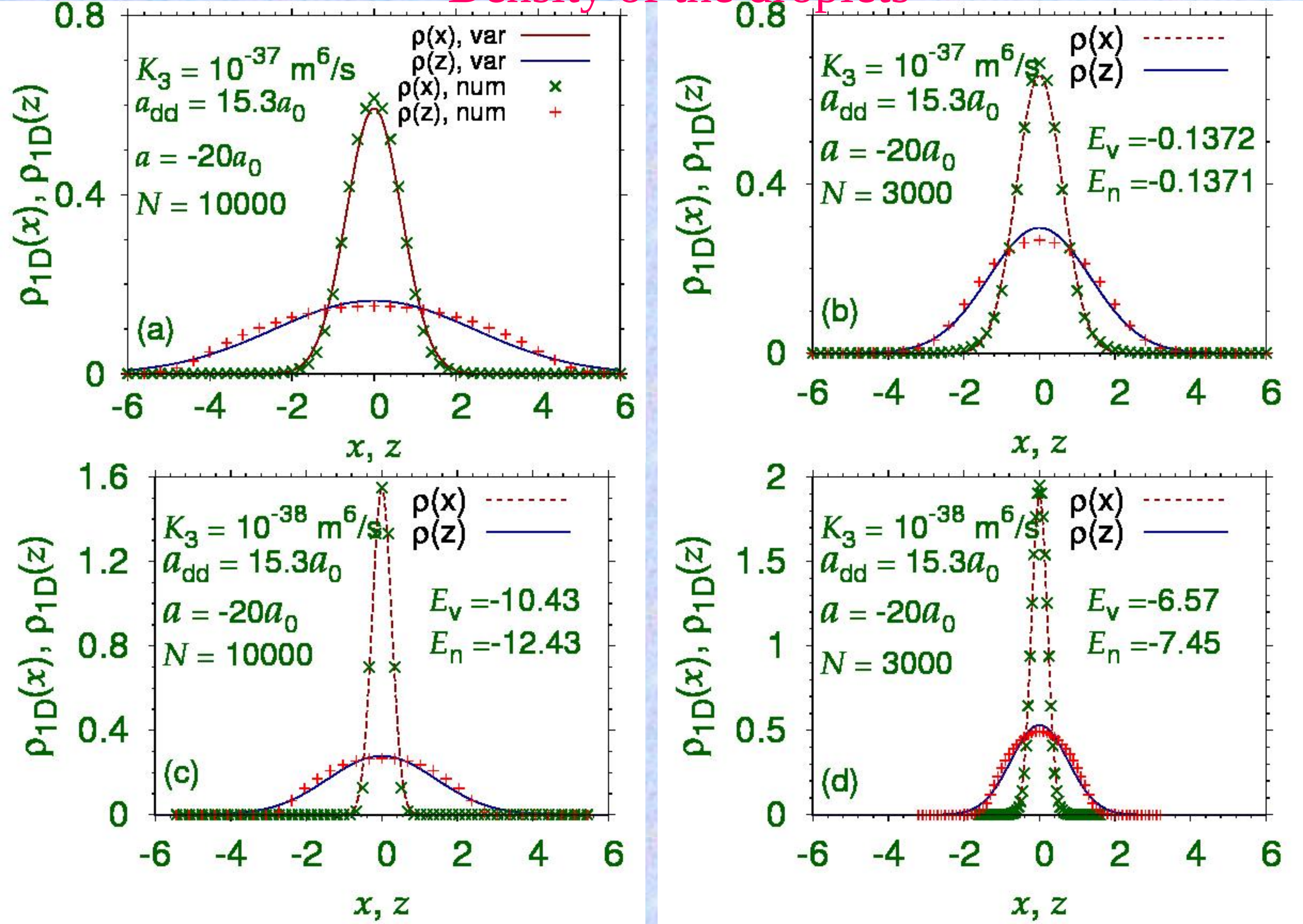
Critical number of atoms



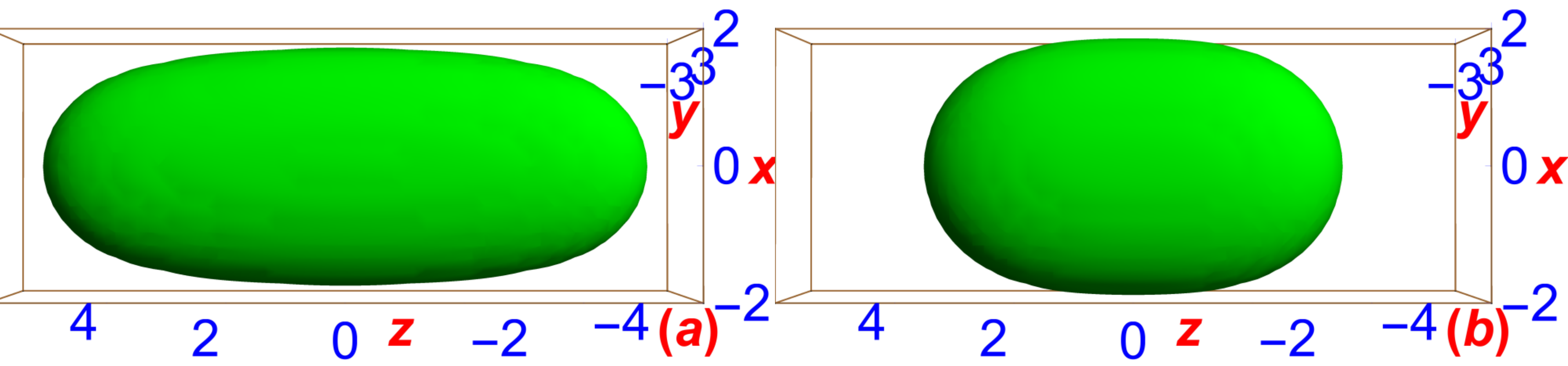
Energy and size of droplets



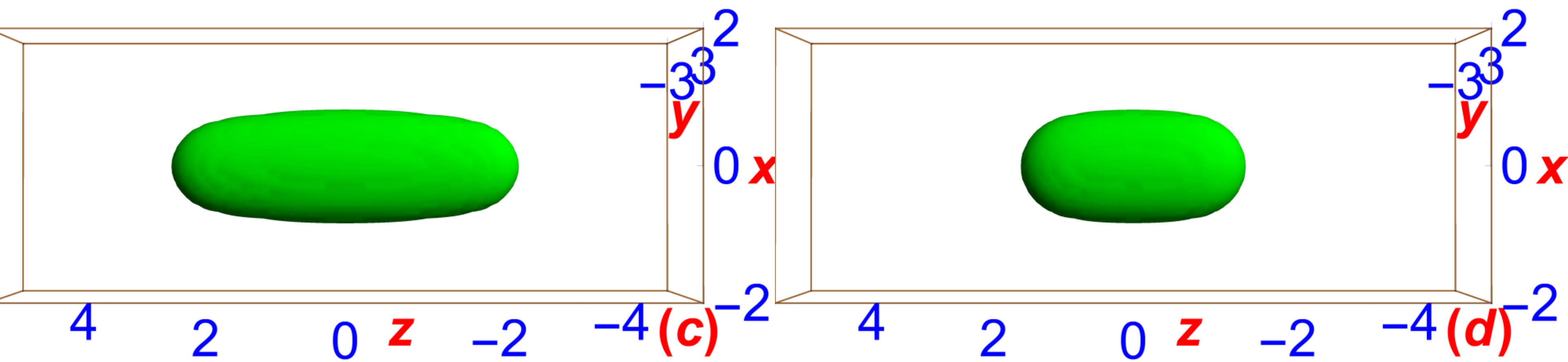
Density of the droplets



3D Isodensity contour of the QB

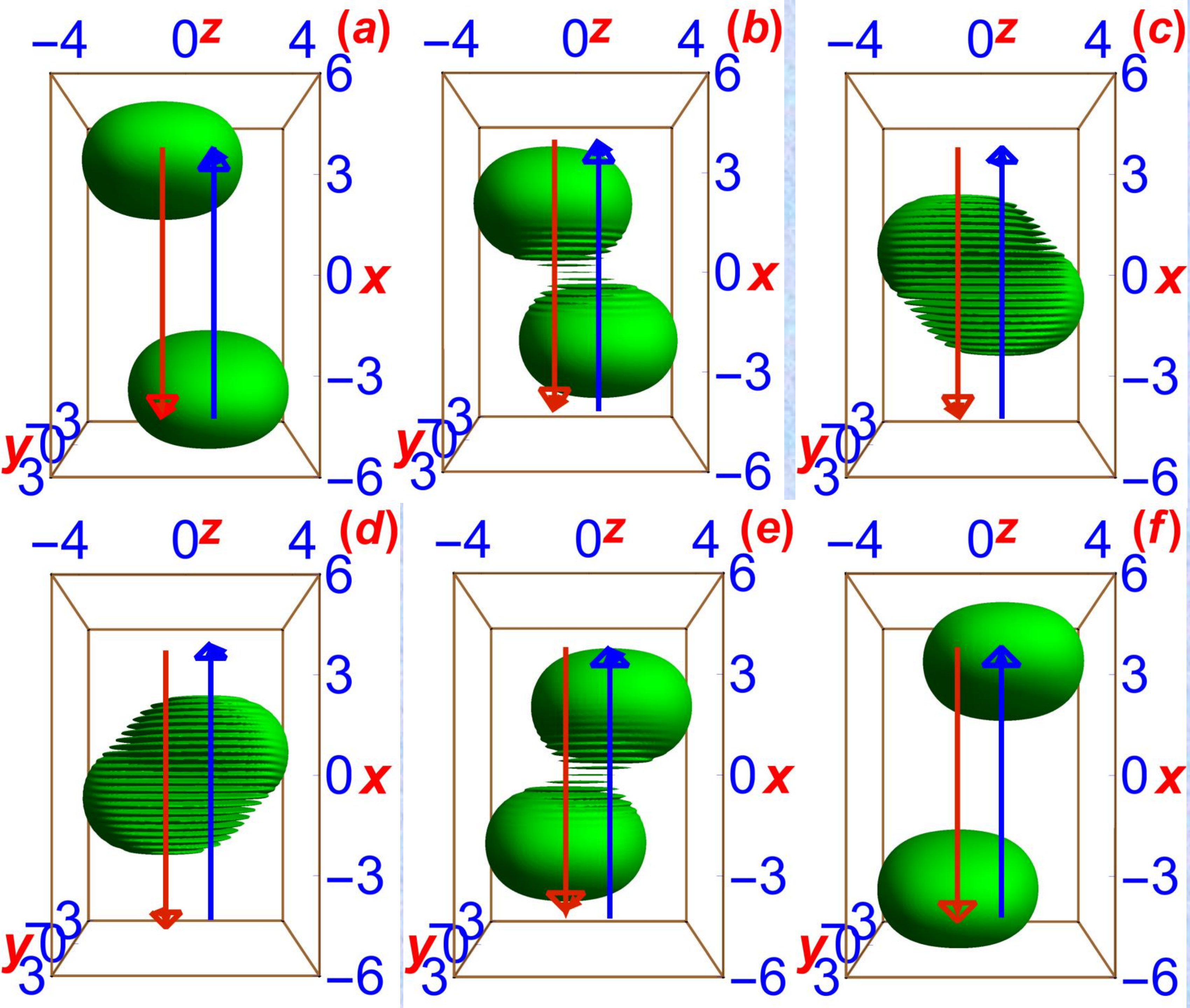


^{52}Cr QB with $a = -20a_0$ for: (a) $N = 10000$, $K_3 = 10^{-37}$ m^6/s, (b) $N = 3000$, $K_3 = 10^{-37}$ m^6/s, (c) $N = 10000$, $K_3 = 10^{-38}$ m^6/s, and (d) $N = 3000$, $K_3 = 10^{-38}$ m^6/s.

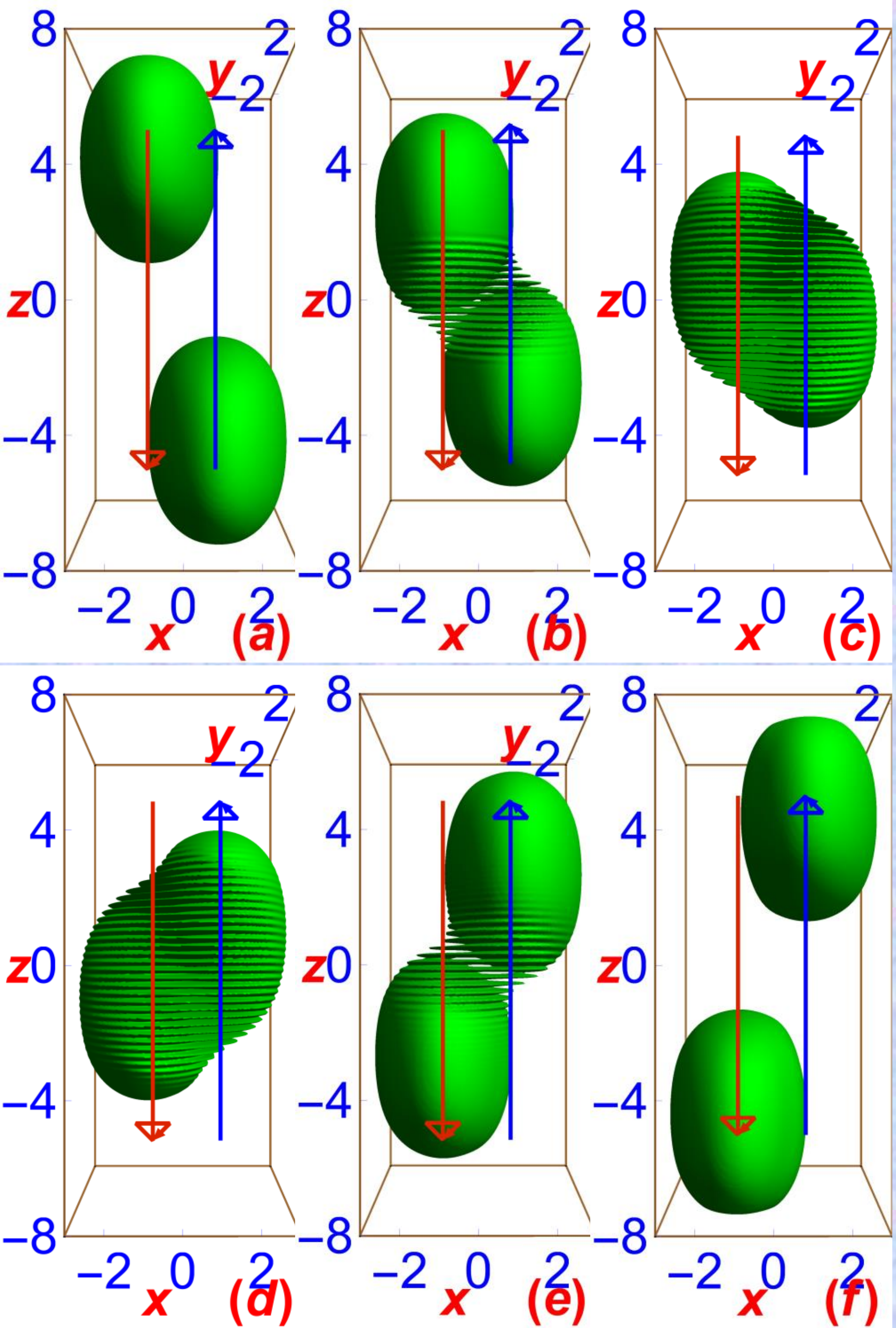


QB-QB Collision

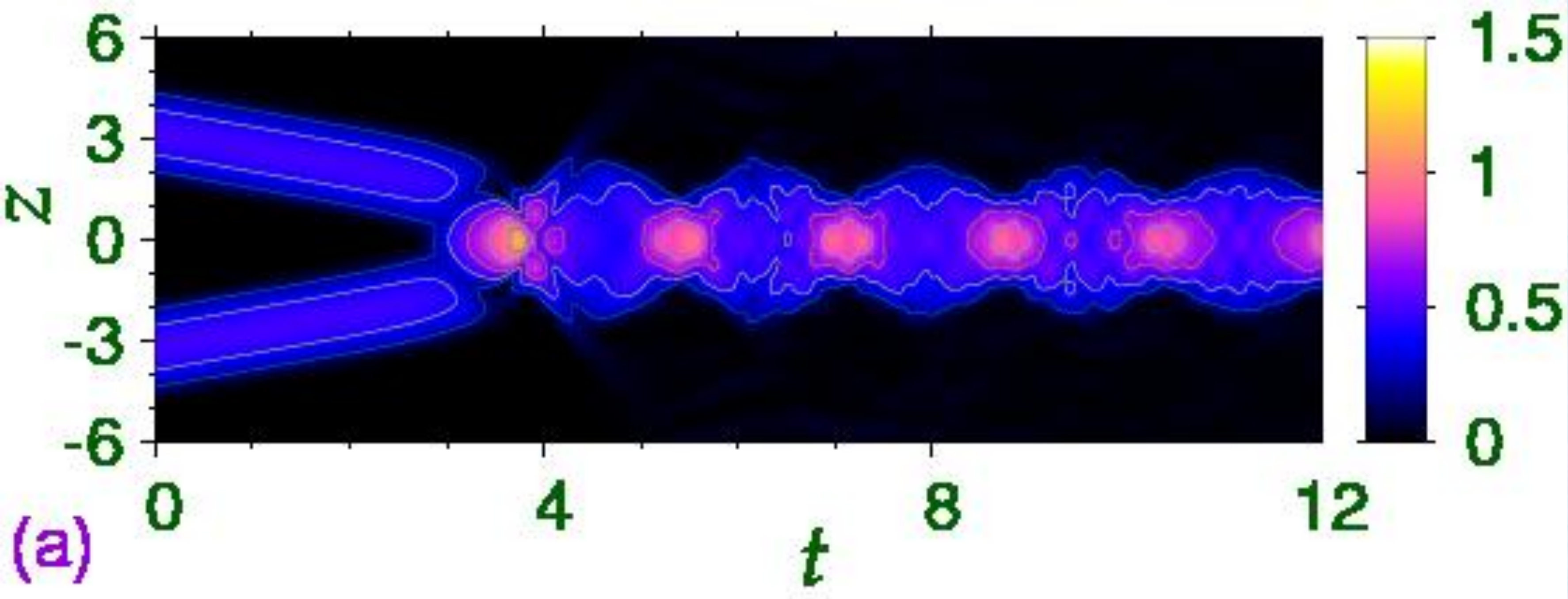
- Numerical simulation in 3D demonstrates quasi-elastic frontal collision at high velocities.
- Molecule formation at low velocities along polarization direction z.
- Bouncing back at low velocities along direction x.



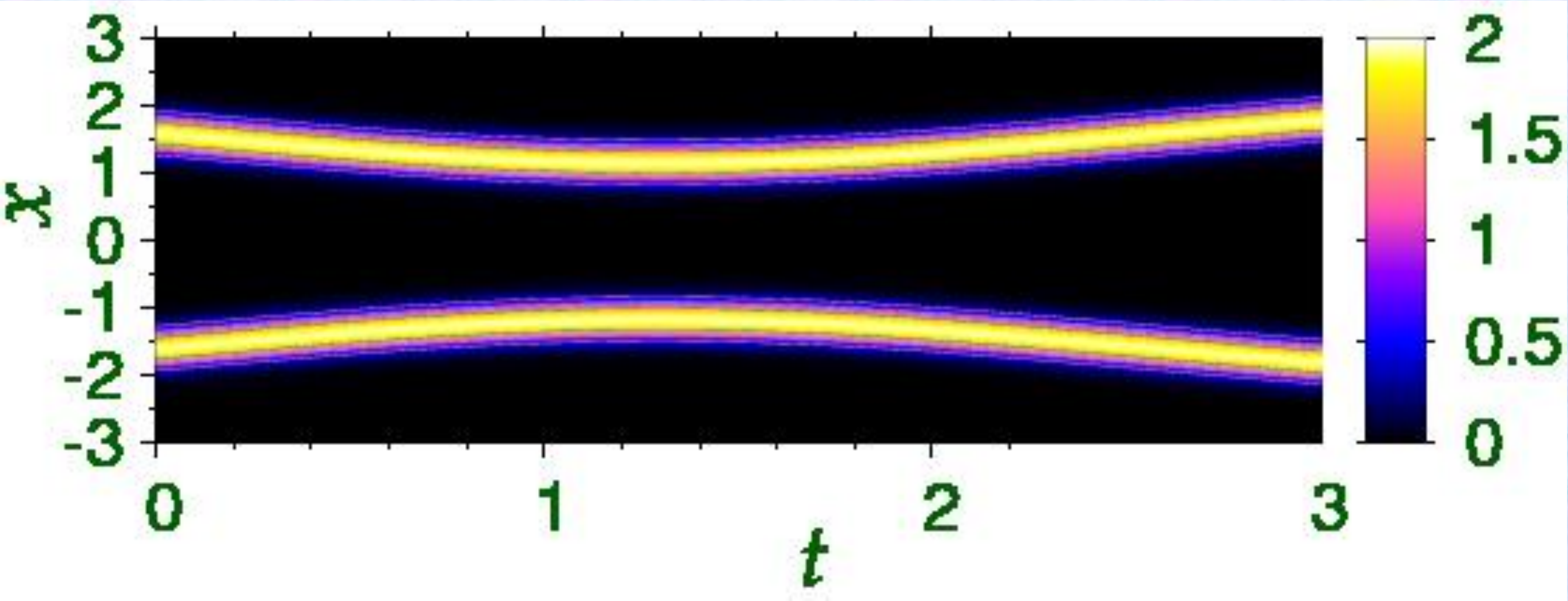
$\nu=38$



v=38



v=0.5



Boson-fermion quantum ball

- Repulsive or attractive boson-boson interaction and fermion-fermion Pauli repulsion
- Attractive boson-fermion interaction
- A repulsive three-boson interaction and/or LHY correction for repulsive two-boson interaction will stop collapse

Trapped boson-fermion mixture:

$$[-\frac{\hbar^2}{2m_1}\nabla^2 + V + \frac{4\pi\hbar^2 a_1 N_1}{m_1}|\psi_1|^2 + \frac{2\hbar^2}{m_1}\alpha\pi a_1{}^{5/2} N_1{}^{3/2}|\psi_1|^3$$

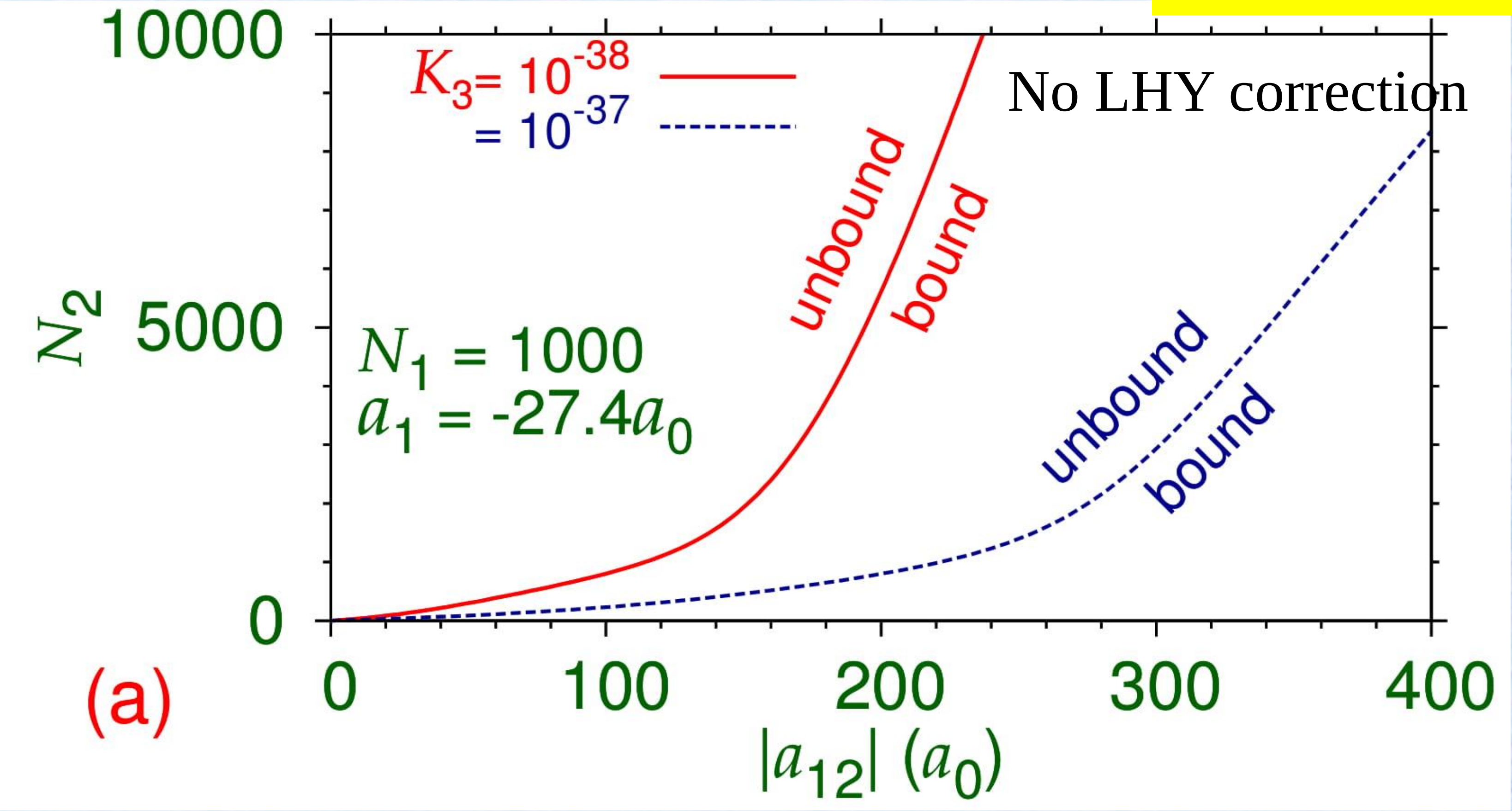
$$+\frac{\hbar N_1{}^2 K_3}{2}|\psi_1|^4 + \frac{2\pi\hbar^2 a_{12} N_2}{m_R}|\psi_2|^2]\ \psi_1(\mathrm{r},t) = i\hbar\frac{\partial}{\partial t}\psi_1(\mathrm{r},t)$$

$$[-\frac{\hbar^2}{8m_2}\nabla^2 + V + \frac{\hbar^2(3\pi^2 N_2|\psi_2|^2)^{2/3}}{2m_2}$$

$$+\frac{2\pi\hbar^2 a_{12} N_1}{m_R}|\psi_1|^2]\ \psi_2(\mathrm{r},t) = i\hbar\frac{\partial}{\partial t}\psi_2(\mathrm{r},t)$$

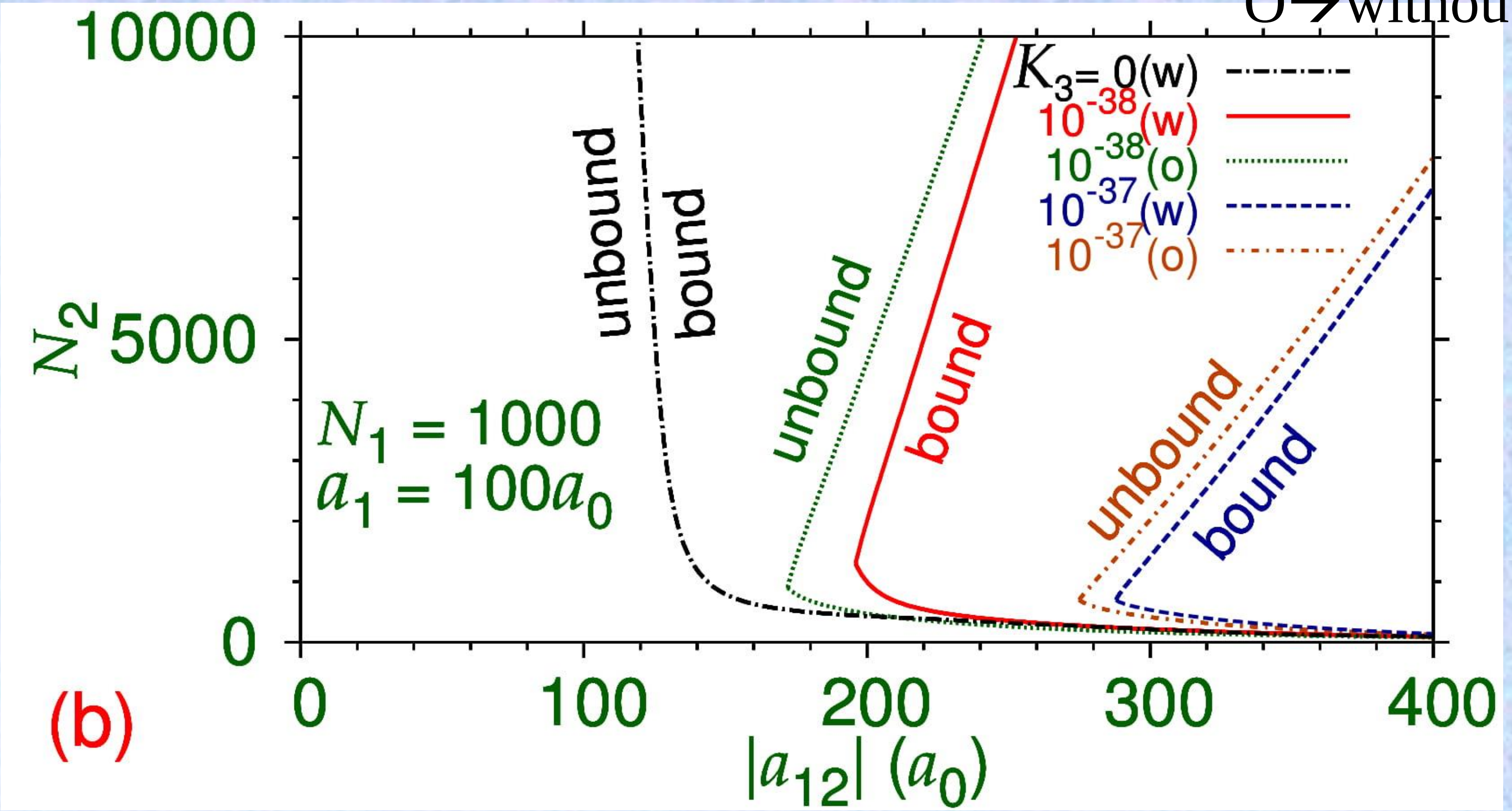
Boson-fermion quantum ball for attractive boson-boson interaction

^{7}Li-^{6}Li mixure



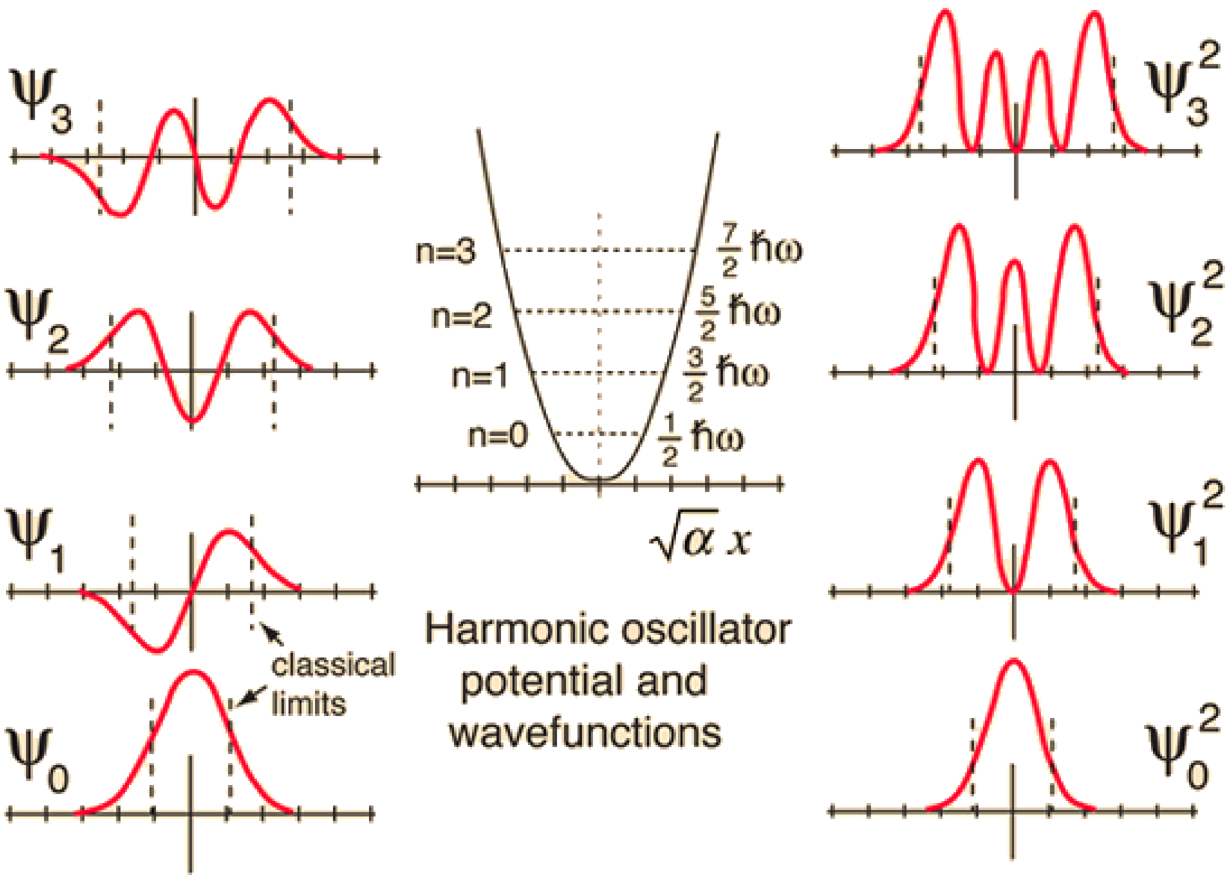
Boson-fermion quantum ball for repulsive boson-boson interaction

W→ with LHY
O→without

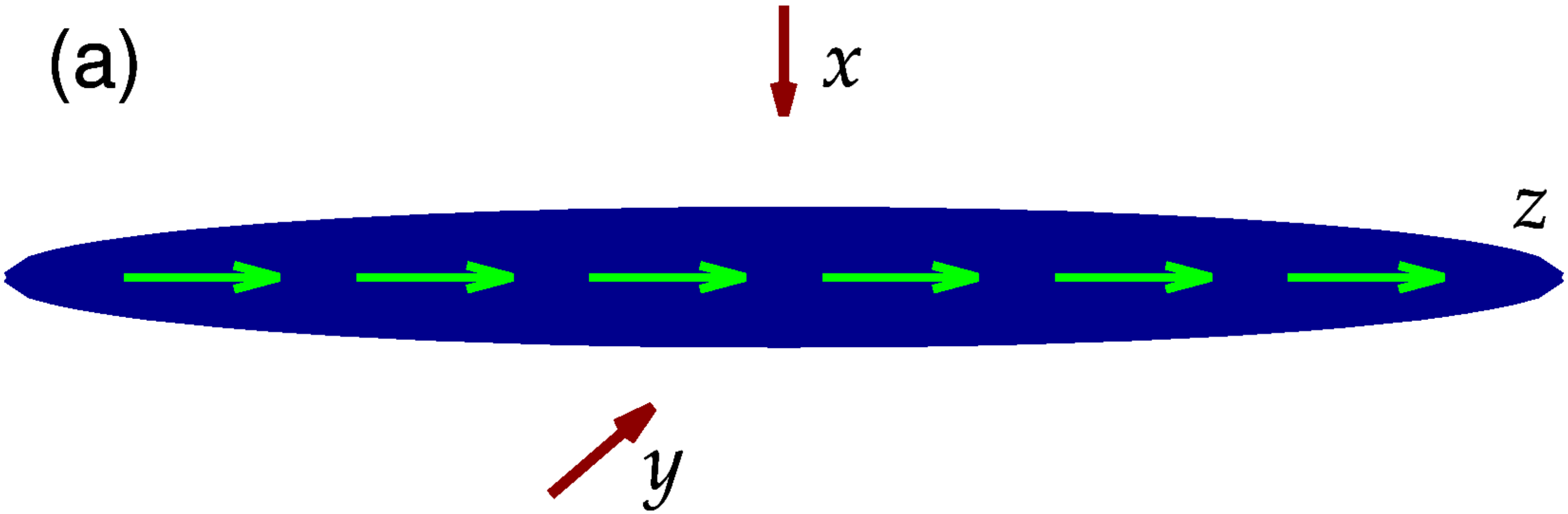


One-dimensional dark soliton

- Like the first excited state of harmonic oscillator



1D dipolar solitons with repulsive contact interactions



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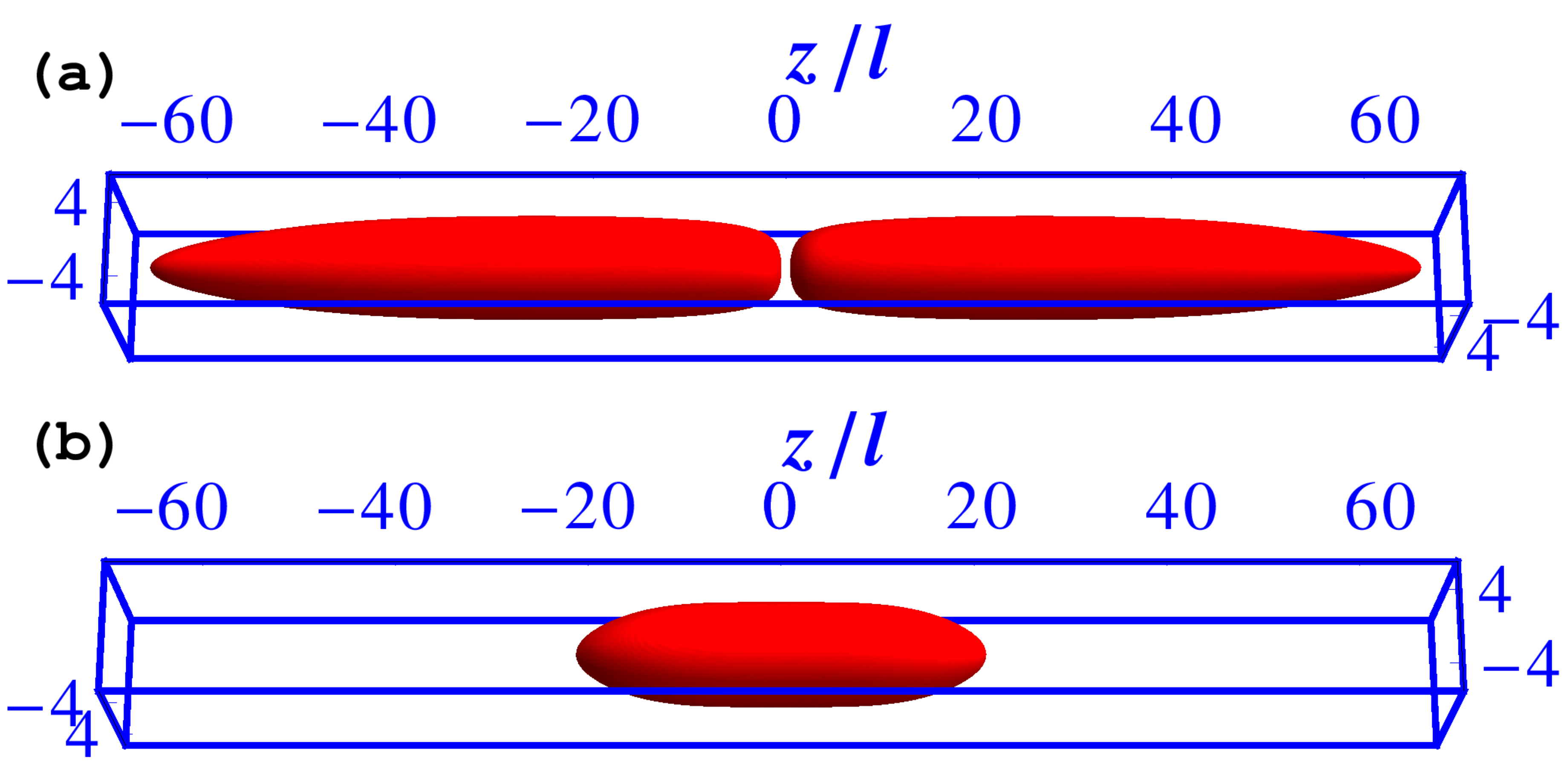
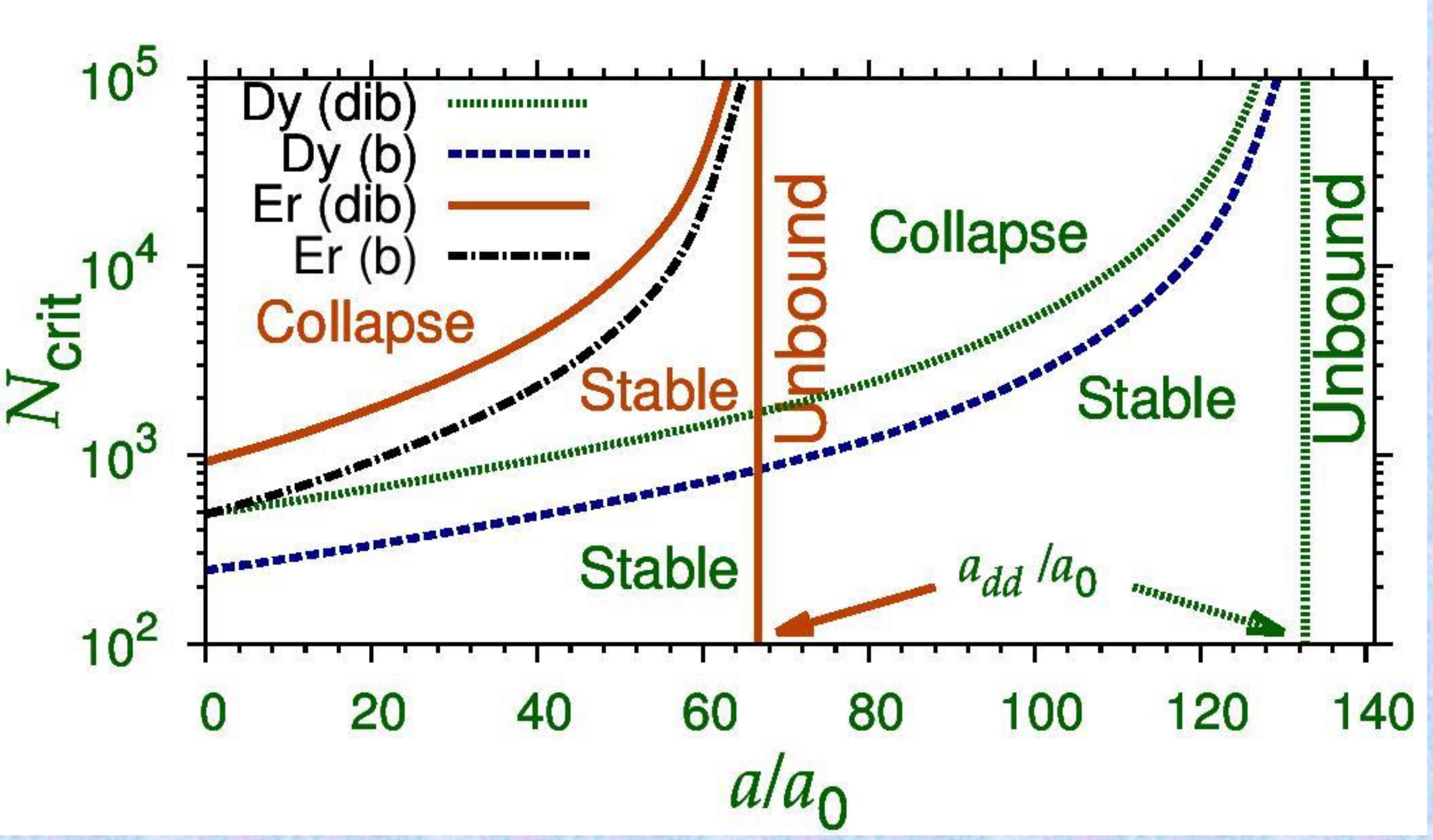
Stable dark soliton of a dipolar BEC

1000 ^{164}Dy atoms $a_{dd} = 132.7 a_0$, $a = 80 a_0$, $l = 1\mu m$

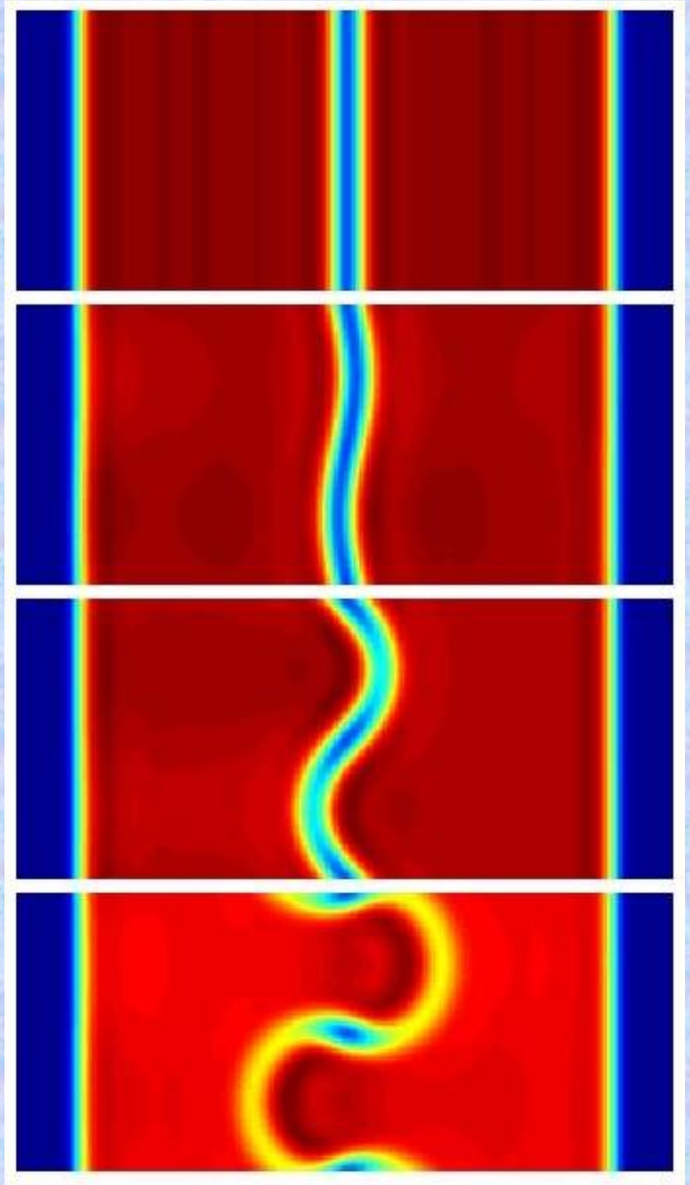


SKA Phys. Rev. A 89 (2014) 043615

$a_{dd}(\mathrm{Dy})=132.7a_0$, $a_{dd}(\mathrm{Er})=66.7a_0$

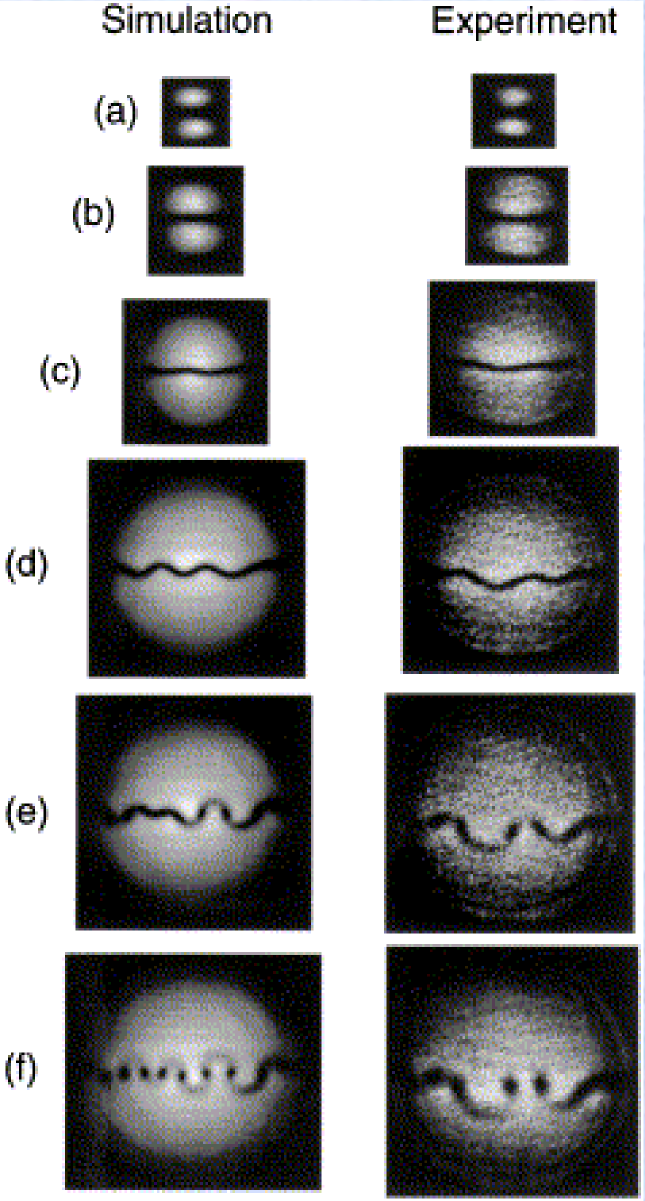


Snake instability of dark soliton in a fermion gas



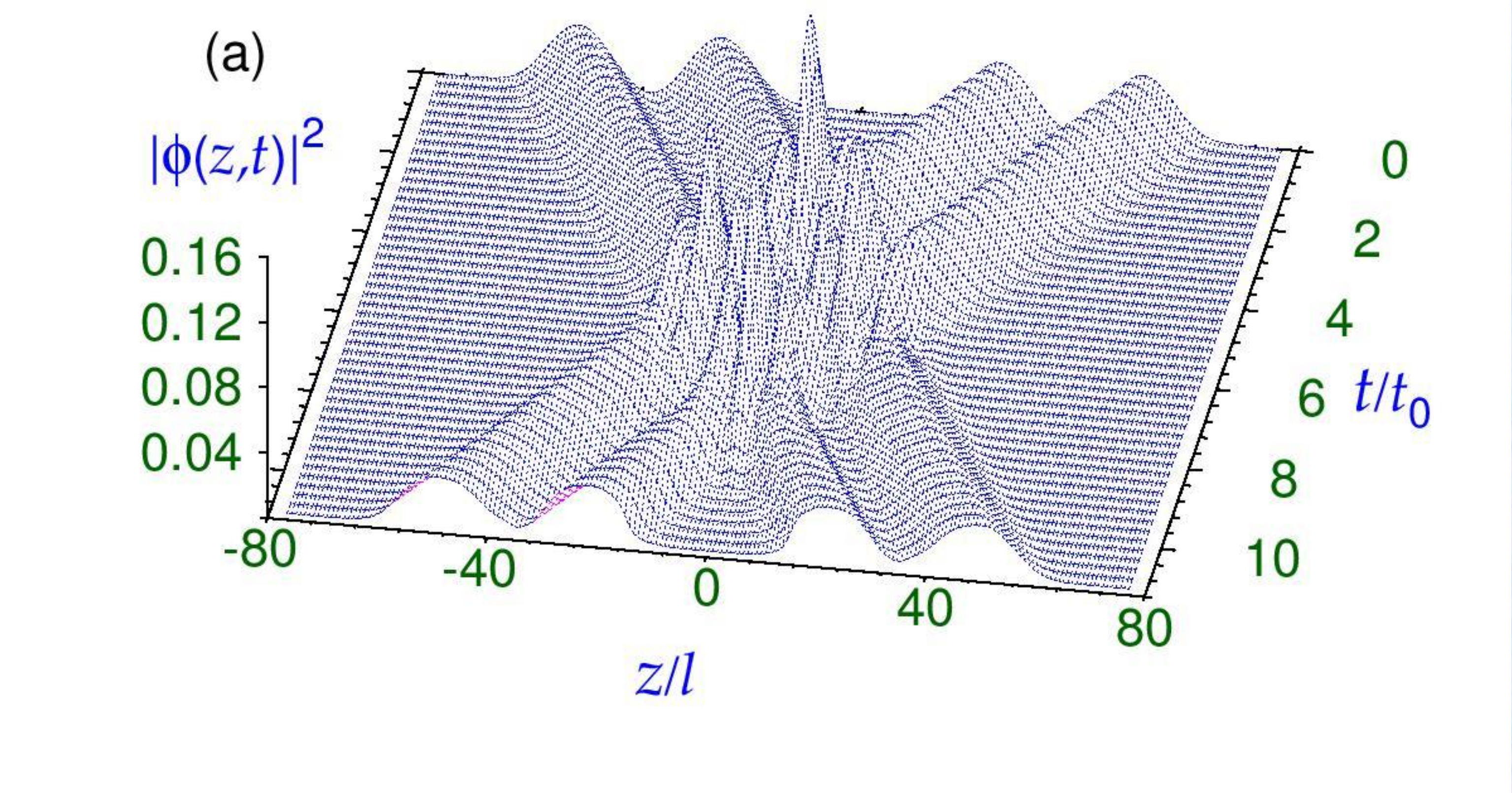
Pitaevskii, Trento

Snake instability of dark optical soliton



Yuri Kivshar, Canberra

Collision of two dark-in-bright stable dipolar solitons



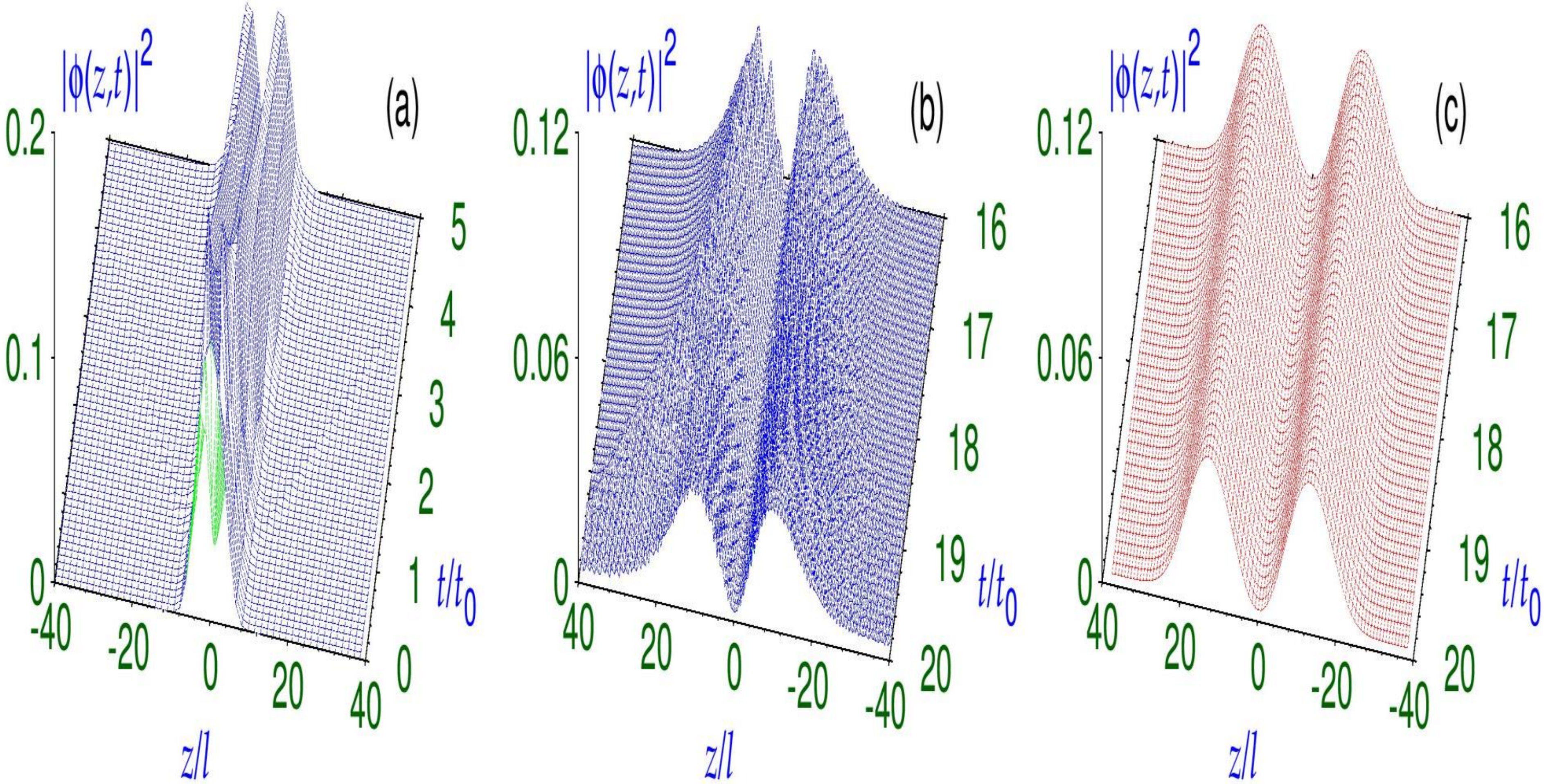
Velocity=2.4 mm/s, t_0 = 2.6ms,

1000 ^{164}Dy atoms, $a_{dd} = 132.7\ a_0$

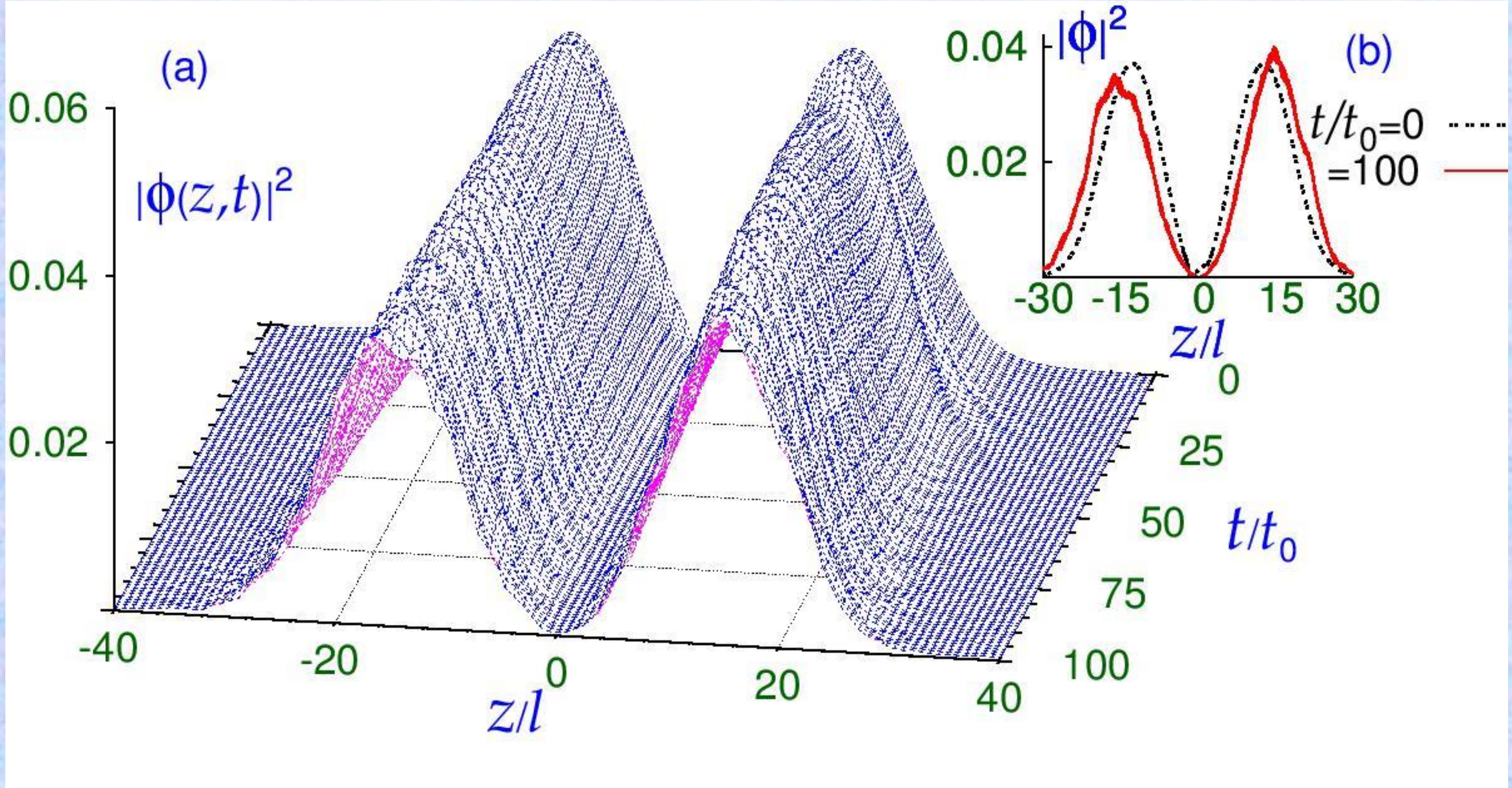
$a = 80a_0$, $l = 1\mu$m

Create a dark soliton in a laboratory:

From a phase-imprinted bright soliton of 1000 ^{164}Dy atoms with $a = 80a_0$ and $l = 1$ μm



Stability of dark-in-bright soliton of 1000 ^{164}Dy atoms with a = 80a_0. The initial state was moved to z = -2 μm.



Unitarity achieved in BEC

P. Makotyn, C. E. Klauss, D. L. Goldberger, E. A. Cornell, and D. S. Jin, Nature Phys. 10, 116 (2014)

"We present time-resolved measurements of the momentum distribution of a Bose-condensed gas that is suddenly jumped to unitarity,
where contrary to expectation, we observe that the gas lives long enough to permit the momentum to evolve to a quasi-steady-state distribution, consistent with universality, while remaining degenerate."

Rapidly rotating BEC

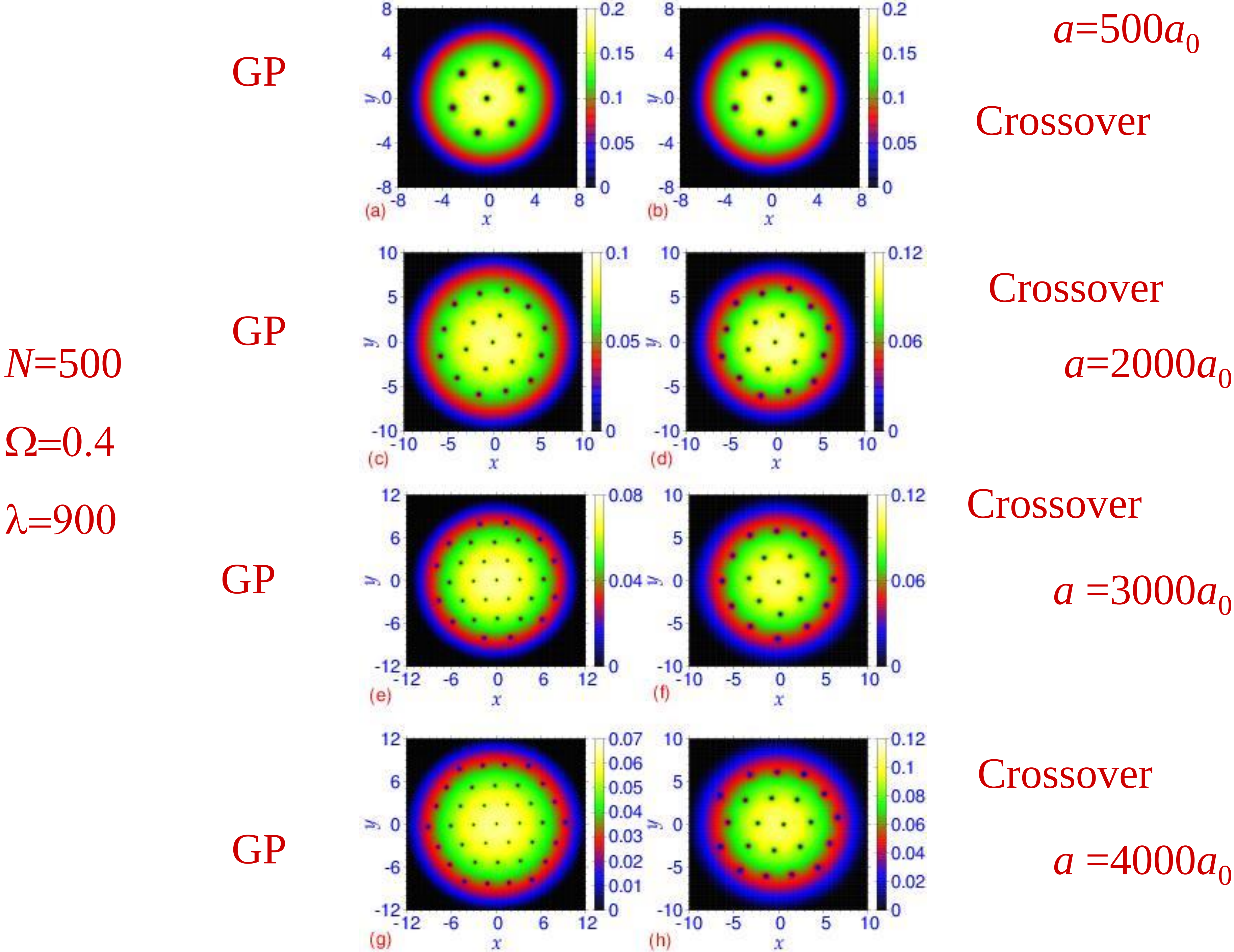
Rotating BEC, vortex-lattice formation dynamics. We assume that the trap rotates with a fixed angular frequency around *z* axis.

$$\left[-\frac{1}{2}\nabla_{\rho}{}^{2}-\frac{1}{2}\frac{\partial^2}{\partial z^2}+\frac{1}{2}\left(\gamma\rho^2+\lambda z^2\right)-\ell_z\Omega+4\pi aN\,|\,\psi\,|^2\right]\psi=i\frac{\partial\psi}{\partial t}$$

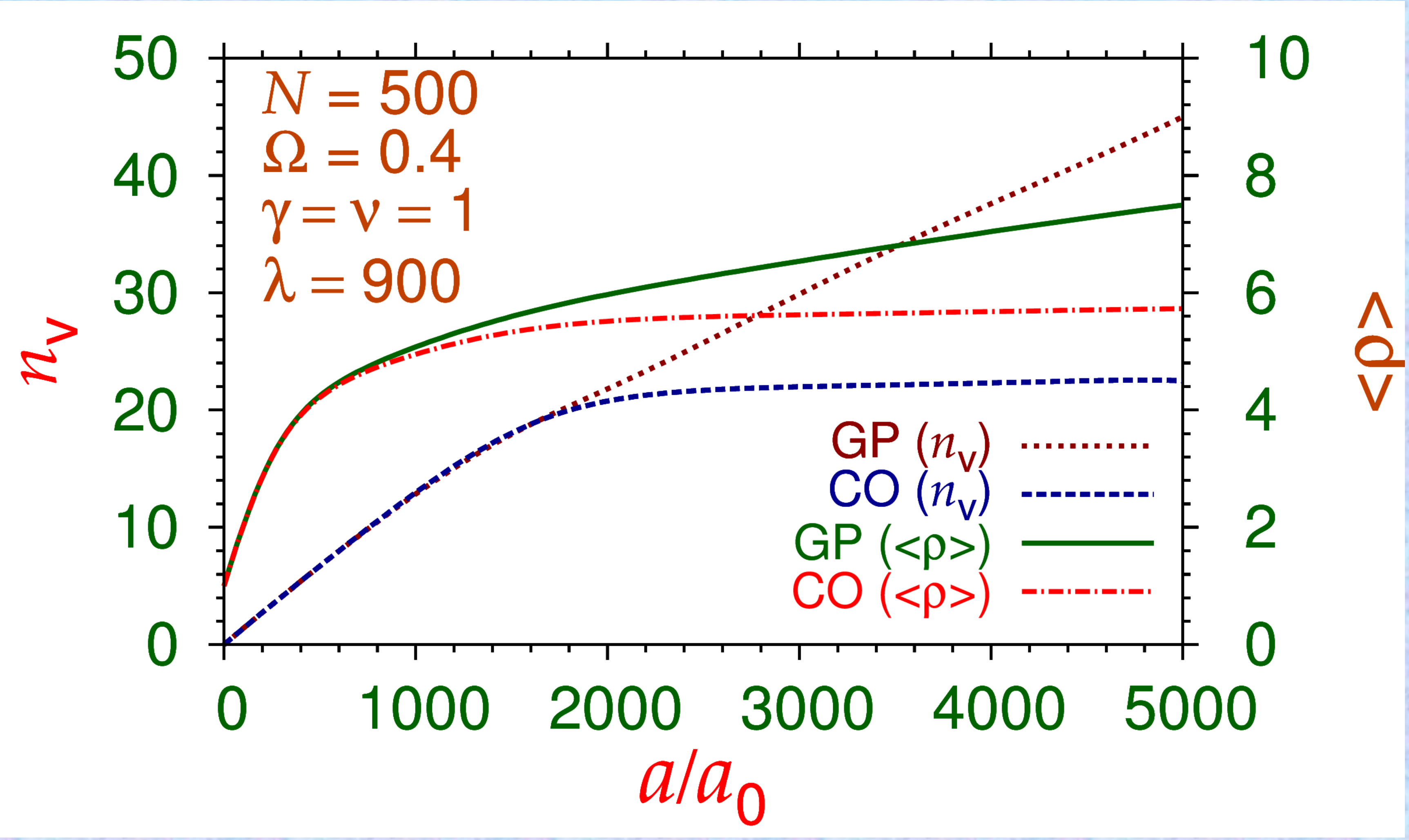
We assume that the trap rotates with a fixed angular frequency Ω around *z* axis. In the rotating frame the original Hamiltonian changes to $H' = H - \ell_z\Omega$, viz. Landau+Lifshitz, Mechanics, where

$$\ell_z = -i\left(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial z}\right).$$

Vortex Lattice in weak-coupling to unitarity crossover



GP
Crossover
a=500a0
N=500
Ω=0.4
λ=900
Crossover
a=2000a0
GP
Crossover
a =3000a0
GP
Crossover
a =4000a0
GP
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
x
y



N = 500
Ω = 0.4
γ = ν = 1
λ = 900
GP (n_v)
CO (n_v)
GP (<ρ>)
CO (<ρ>)
n_v
<ρ>
a/a_0
0
1000
2000
3000
4000
5000
0
10
20
30
40
50
0
2
4
6
8
10

Concluding remarks

- I. A quantum ball (dipolar, boson, boson-boson, boson-fermion) can be stabilized for a small repulsive three-body interaction and/or LHY correction
- II. Robust stable dark soliton in dipolar BEC
- III. Vortex lattice in BEC at unitarity
- IV. Further experiments expected in the future

- Thank you for your attention